UNIVERSITAS KOMPUTER INDONESIA

Dr. Ir. Yeffry Handoko Putra, M.T

Transfer Function

Transfer Function, Poles and zeros

- For zero initial condition, the differential equation:

$$a_0 y + a_1 \frac{dy}{dt} + \ldots + a_n \frac{d^n y}{dt^n} = b_0 x + b_1 \frac{dx}{dt} + \ldots + b_m \frac{d^m x}{dt^m}$$

$$a_0 Y(s) + a_1 s Y(s) + \ldots + a_n s^n Y(s) = b_0 X(s) + b_1 s X(s) + \ldots + b_m s^m X(s)$$

$$A(s)Y(s) = B(s)Y(s)$$

Can be transformed into Transfer Function

$$H(s) = \frac{Y(s)}{X(s)} = \frac{B(s)}{A(s)} = \frac{b_0 + b_1 s + \ldots + b_m s^m}{a_0 + a_1 s + \ldots + a_n s^n}$$

Zero:B(s) = 0

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Can be transformed into Transfer Function

$$H(s) = \frac{Y(s)}{X(s)} = \frac{B(s)}{A(s)}$$

$$= \frac{\mathrm{b}_0 + b_1 s + \ldots + b_m s^m}{a_0 + a_1 s + \ldots + a_n s^n}$$

Pole: A(s) = 0

Exercise

- Find pole and zeros from this transfer function:

$$H(s) = \frac{s^2 + 2s + 2}{s^2 + 4s + 13}$$

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❖ Find pole and zeros from this transfer function:

$$H(s) = \frac{s^2 + 2s + 2}{s^2 + 4s + 13}$$

zero : $s^2+2s+2=0$

$$s = \frac{-2 \pm \sqrt{2^2 - 4(2)}}{2}$$
$$= -1 \pm j$$

Exercise

❖ Find pole and zeros from this transfer function:

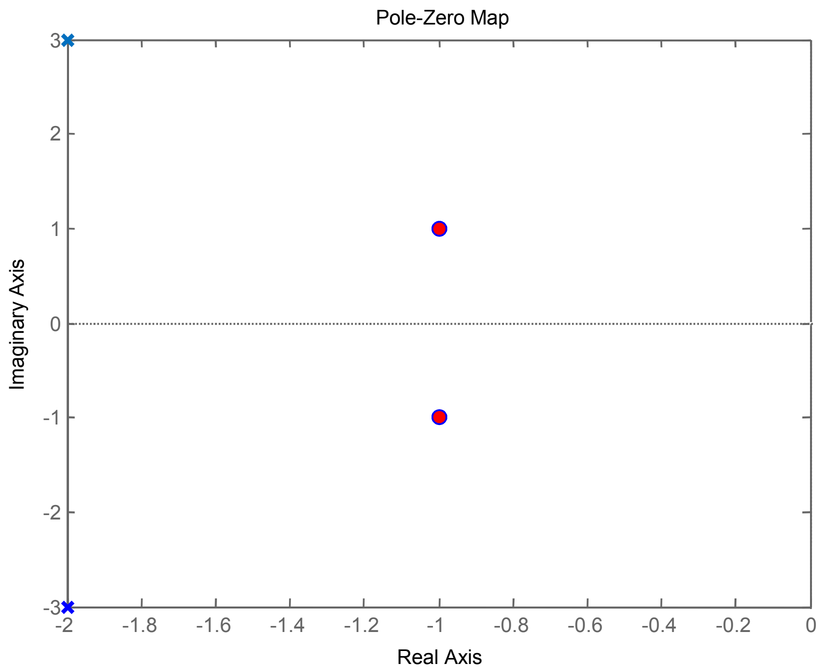
$$H(s) = \frac{s^2 + 2s + 2}{s^2 + 4s + 13}$$

Pole : $s^2+4s+13=0$

$$s = -2 \pm j3$$

Exercise

- S-plane:



Exercise

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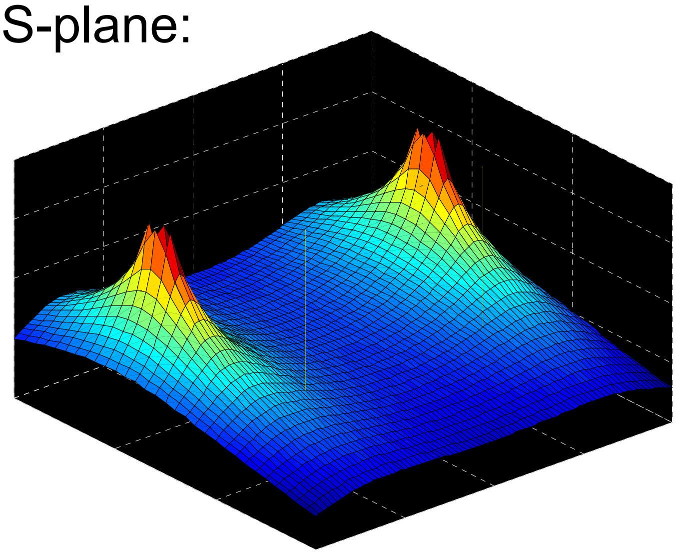


Frequency Response of single pole system

- Frequency response H(ω) is found by substituted jω for s
- Frequency response for

$$H(s) = \frac{1}{s+7}$$

is

$$H(\omega) = \frac{1}{j\omega + 7}$$

Frequency Response from pole/zero diagram

- If transfer function H(s) has *m* zero and *n* pole, it can be written as:

$$H(s) = \frac{A(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

The Frequency response:

$$H(\omega) = \frac{A(j\omega-z_1)(j\omega-z_2)\ldots(j\omega-z_m)}{(j\omega-p_1)(j\omega-p_2)\ldots(j\omega-p_n)}$$

Frequency Response from pole/zero diagram

- Magnitude of H(ω):

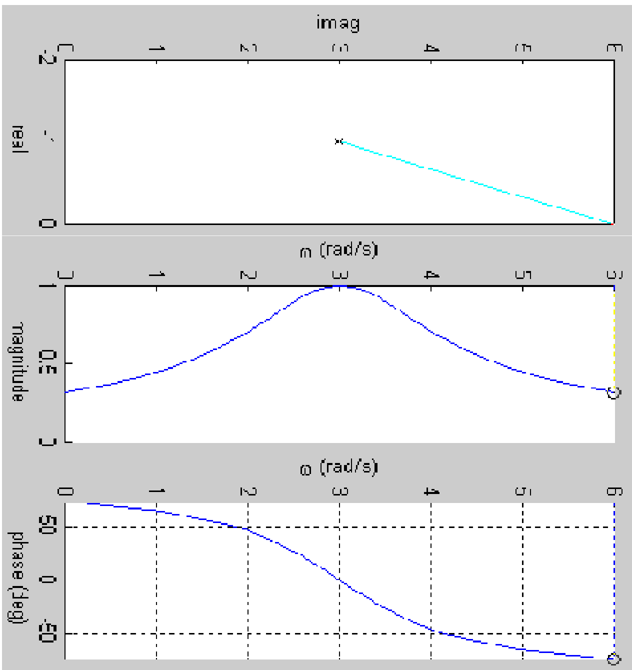
$$|H(\omega)| = \frac{A|j\omega - z_1||j\omega - z_2|\ldots|j\omega - z_m|}{|j\omega - p_1||j\omega - p_2|\ldots|j\omega - p_n|}$$

Phase of H(ω):

$$\begin{aligned}\angle H(\omega) &= \angle(j\omega - z_1) + \angle(j\omega - z_2) + \ldots + \angle(j\omega - z_m) \\ &\quad - \angle(j\omega - p_1) - \angle(j\omega - p_2) - \ldots - \angle(j\omega - p_n)\end{aligned}$$

Frequency Response of single pole

$$H(j\omega) = \frac{1}{(j\omega - p_1)}$$



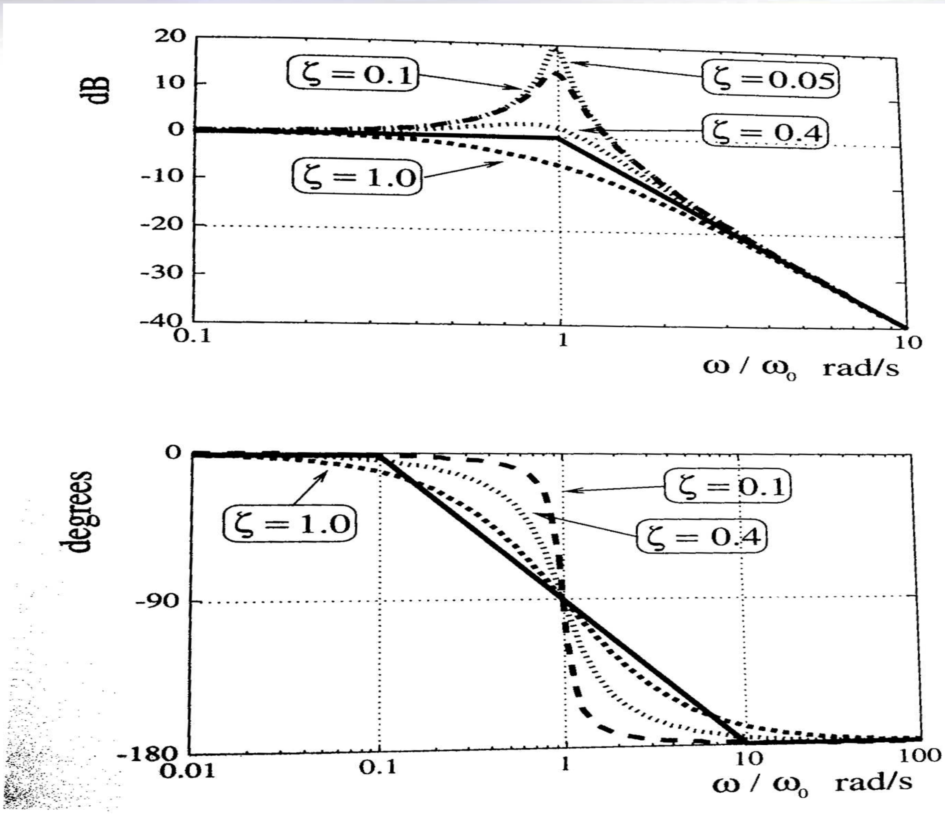
Bode Plot

- Write denumerator in the context of bode plot

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = \omega_0^2((s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1)$$

- Magnitude in logarithmic in decibels (dB):20log|H(ω)|

Bode Plot



Example Bode Plot

- Sketch bode plot for following transfer function

$$H(s) = \frac{s+20}{s+2000}$$

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The frequency Response

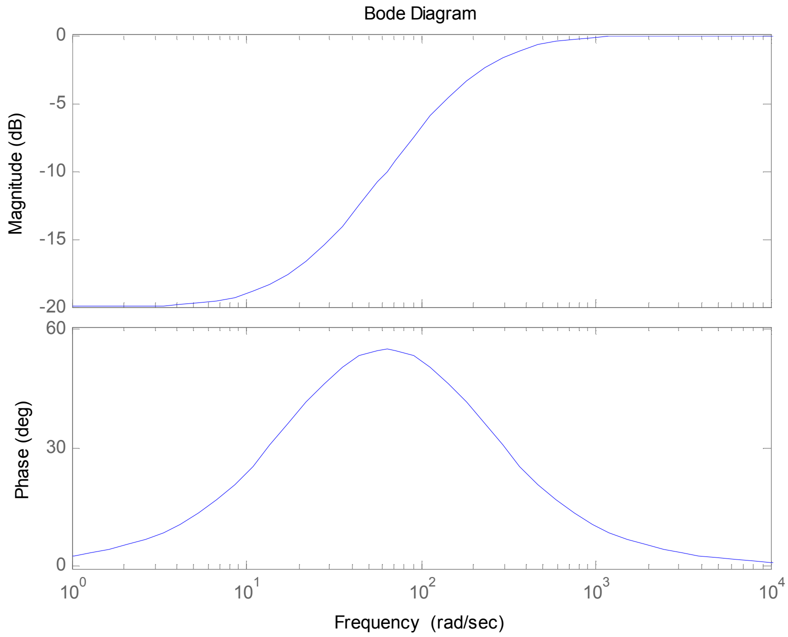
$$H(j\omega) = \frac{j\omega+20}{j\omega+2000}$$

$$= \frac{\mathrm{j}\omega/20+1}{(j\omega/2000+1)100}$$

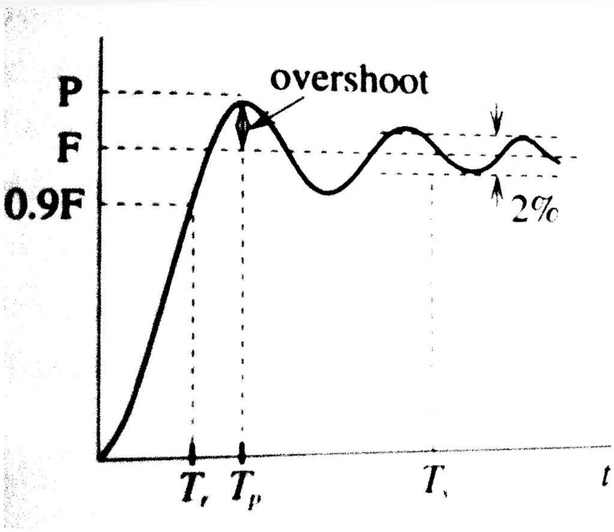
Example Bode Plot

❖ Sketch bode plot for following transfer function

$$H(s) = \frac{s+20}{s+2000}$$



Rise Time and Bandwidth



Rise time:

$T_r = \tau \ln(10)$ s

τ = Time Constant

Bandwidth:

$\omega_B \cong \omega_0 = 1/\tau$

Peak Time :

$$T_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$$

How to Reduce Rise Time?