Some familiarity with programming concepts is assumed, but no knowledge of any particular programming language is needed as a prerequisite. In addition, familiarity with the concepts of statistics and probability theory will be helpful, although two chapters review the required theory. Many areas of application are discussed, but the technical details are kept simple so that the book will be useful to students of different backgrounds. A course based on the text could be introduced at the senior undergraduate or graduate level in many disciplines.

The first chapter discusses the types of models that are basic to any simulation. The second chapter considers the topic of organizing a system study. The nature of the simulation technique is then introduced in Chapter 3. Continuous-system simulation is examined in Chapter 4, and its application to System Dynamics studies is illustrated in Chapter 5. Chapters 6 and 7 are the ones that introduce the necessary statistics and probability theory.

The application of discrete-system simulation is demonstrated in Chapter 8, using a hand-worked example of a simple telephone-system. Two chapters are then devoted to GPSS, followed by two chapters describing SIMSCRIPT. In both cases, the first of the two chapters is self-contained, and would be sufficient for students needing only a brief introduction. The second chapter in each case includes, as a worked-out example, the same telephone-system problem presented in Chapter 8, thus giving an opportunity to compare the three approaches.

Chapter 13 discusses the programming techniques involved in the design and construction of simulation programming systems. Its prime interest will be to students concerned with the evaluation and design of simulation languages. The last chapter, also, is of a more technical nature, since it discusses the topic of analyzing statistical outputs of simulation runs.

I would like to thank CACI, Inc., for providing the SIMSCRIPT listings used in Chapters 11 and 12, and Dr. E. C. Russell for his generous help in both correcting the SIMSCRIPT examples and reviewing those chapters.

GEOFFREY GORDON

SYSTEM MODELS

1-1

The Concepts of a System

The term *system* is used in such a wide variety of ways that it is difficult to produce a definition broad enough to cover the many uses and, at the same time, concise enough to serve a useful purpose, (6), (12), and (20).[1] We begin, therefore, with a simple definition of a system and expand upon it by introducing some of the terms that are commonly used when discussing systems. A *system* is defined as an aggregation or assemblage of objects joined in some regular interaction or interdependence. While this definition is broad enough to include static systems, the principal interest will be in dynamic systems where the interactions cause changes over time.

As an example of a conceptually simple system, consider an aircraft flying under the control of an autopilot (see Fig. 1-1). A gyroscope in the autopilot detects the difference between the actual heading and the desired heading. It sends a signal to move the control surfaces. In response to the control surface movement, the airframe steers toward the desired heading.

As a second example, consider a factory that makes and assembles parts into a product (see Fig. 1-2). Two major components of the system are the fabrication

[1] Parenthetical numbers in text refer to items in bibliography at end of chapter.

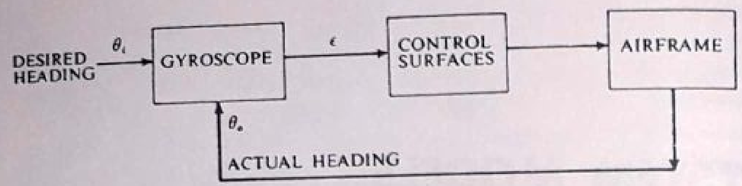


Figure 1-1. An aircraft under autopilot control.

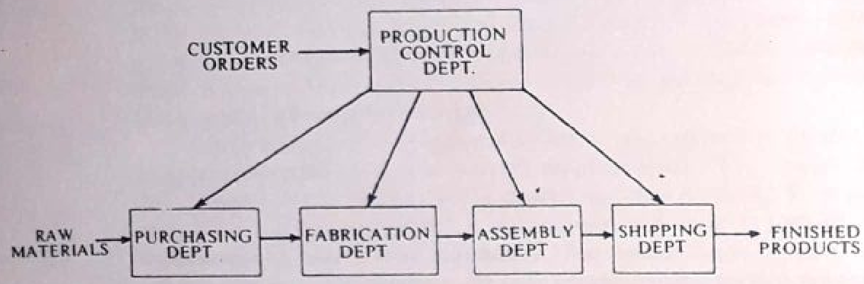


Figure 1-2. A factory system.

department making the parts and the assembly department producing the products. A purchasing department maintains a supply of raw materials and a shipping department dispatches the finished products. A production control department receives orders and assigns work to the other departments.

In looking at these systems, we see that there are certain distinct objects, each of which possesses properties of interest. There are also certain interactions occurring in the system that cause changes in the system. The term *entity* will be used to denote an object of interest in a system; the term *attribute* will denote a property of an entity. There can, of course, be many attributes to a given entity. Any process that causes changes in the system will be called an *activity*. The term *state of the system* will be used to mean a description of all the entities, attributes, and activities as they exist at one point in time. The progress of the system is studied by following the changes in the state of the system.

In the description of the aircraft system, the entities of the system are the airframe, the control surfaces, and the gyroscope. Their attributes are such factors as speed, control surface angle, and gyroscope setting. The activities are the driving of the control surfaces and the response of the airframe to the control surface movements. In the factory system, the entities are the departments, orders, parts, and products. The activities are the manufacturing processes of the departments.

Attributes are such factors as the quantities for each order, type of part, or number of machines in a department.

Figure 1-3 lists examples of what might be considered entities, attributes, and activities for a number of other systems. If we consider the movement of cars as a traffic system, the individual cars are regarded as entities, each having as attributes its speed and distance traveled. Among the activities is the driving of a car. In the case of a bank system, the customers of the bank are entities with the balances of their accounts and their credit statuses as attributes. A typical activity would be the action of making a deposit. Other examples are shown in Fig. 1-3.

SYSTEM	ENTITIES	ATTRIBUTES	ACTIVITIES
TRAFFIC	CARS	SPEED DISTANCE	DRIVING
BANK	CUSTOMERS	BALANCE CREDIT STATUS	DEPOSITING
COMMUNICATIONS	MESSAGES	LENGTH PRIORITY	TRANSMITTING
SUPERMARKET	CUSTOMERS	SHOPPING LIST	CHECKING OUT

Figure 1-3. Examples of systems.

The figure does not show a complete list of all entities, attributes, and activities for the systems. In fact, a complete list cannot be made without knowing the purpose of the system description. Depending upon that purpose, various aspects of the system will be of interest and will determine what needs to be identified.

1-2

System Environment

A system is often affected by changes occurring outside the system. Some system activities may also produce changes that do not react on the system. Such changes occurring outside the system are said to occur in the *system environment*. An important step in modeling systems is to decide upon the boundary between the system and its environment. The decision may depend upon the purpose of the study.

In the case of the factory system, for example, the factors controlling the arrival of orders may be considered to be outside the influence of the factory and therefore part of the environment. However, if the effect of supply on demand is to be considered, there will be a relationship between factory output and arrival of orders, and this relationship must be considered an activity of the system. Similarly, in the case of a bank system, there may be a limit on the maximum interest rate that can be paid. For the study of a single bank, this would be regarded as a constraint imposed by the environment. In a study of the effects of monetary laws on the banking industry, however, the setting of the limit would be an activity of the system.

The term *endogenous* is used to describe activities occurring within the system and the term *exogenous* is used to describe activities in the environment that affect the system. A system for which there is no exogenous activity is said to be a *closed* system in contrast to an *open* system which does have exogenous activities.

1-3
Stochastic Activities

One other distinction that needs to be drawn between activities depends upon the manner in which they can be described. Where the outcome of an activity can be described completely in terms of its input, the activity is said to be *deterministic*. Where the effects of the activity vary randomly over various possible outcomes, the activity is said to be *stochastic*.

The randomness of a stochastic activity would seem to imply that the activity is part of the system environment since the exact outcome at any time is not known. However, the random output can often be measured and described in the form of a probability distribution. If, however, the *occurrence* of the activity is random, it will constitute part of the environment. For example, in the case of the factory, the time taken for a machining operation may need to be described by a probability distribution but machining would be considered to be an endogenous activity. On the other hand, there may be power failures at random intervals of time. They would be the result of an exogenous activity.

If an activity is truly stochastic, there is no known explanation for its randomness. Sometimes, however, when it requires too much detail or is just too much trouble to describe an activity fully, the activity is represented as stochastic. For example, in modeling elevator service in a building, the re-entry of people into the elevator, after they have been taken to a floor, could be connected with their having left the elevator, by assigning the time they stay on the floor. In most models, however, leaving and re-entry would be treated as separate stochastic

activities, connected only by the fact that the mean rates at which they transfer people are equal.

Assembling the data for a model will often involve an element of uncertainty that arises from sampling or experimental error. A value for some attribute of a model, which is known to be fixed, must be selected from a number of recorded values that contain random errors. Deciding on the best estimate is a statistical exercise. Usually, an arithmetic average will be considered sufficiently accurate.

1-4
Continuous and Discrete Systems

The aircraft and factory systems used as examples in Sec. 1-1 respond to environmental changes in different ways. The movement of the aircraft occurs smoothly, whereas the changes in the factory occur discontinuously. The ordering of raw materials or the completion of a product, for example, occurs at specific points in time.

Systems such as the aircraft, in which the changes are predominantly smooth, are called *continuous systems*. Systems like the factory, in which changes are predominantly discontinuous, will be called *discrete systems*. Few systems are wholly continuous or discrete. The aircraft, for example, may make discrete adjustments to its trim as altitude changes, while, in the factory example, machining proceeds continuously, even though the start and finish of a job are discrete changes. However, in most systems one type of change predominates, so that systems can usually be classified as being continuous or discrete.

The complete aircraft system might even be regarded as a discrete system. If the purpose of studying the aircraft were to follow its progress along its scheduled route, with a view, perhaps, to studying air traffic problems, there would be no point in following precisely *how* the aircraft turns. It would be sufficiently accurate to treat changes of heading at scheduled turning points as being made instantaneously, and so regard the system as being discrete.

In addition, in the factory system, if the number of parts is sufficiently large, there may be no point in treating the number as a discrete variable. Instead, the number of parts might be represented by a continuous variable with the machining activity controlling the rate at which parts flow from one state to another. This is, in fact, the approach of a modeling technique called System Dynamics, which will be discussed in Chap. 5.

There are also systems that are intrinsically continuous but information about them is only available at discrete points in time. These are called *sampled-data* systems, (15). The study of such systems includes the problem of determining the effects of the discrete sampling, especially when the intention is to control the system on the basis of information gathered by the sampling.

This ambiguity in how a system might be represented illustrates an important point. The description of a system, rather than the nature of the system itself, determines what type of model will be used. A distinction needs to be made because, as will be discussed later, the general programming methods used to simulate continuous and discrete models differ. However, no specific rules can be given as to how a particular system is to be represented. The purpose of the model, coupled with the general principle that a model should not be more complicated than is needed, will determine the level of detail and the accuracy with which a model needs to be developed. Weighing these factors and drawing on the experience of knowledgeable people will decide the type of model that is needed.

1-5 System Modeling

To study a system, it is sometimes possible to experiment with the system itself. The objective of many system studies, however, is to predict how a system will perform before it is built. Clearly, it is not feasible to experiment with a system while it is in this hypothetical form. An alternative that is sometimes used is to construct a number of prototypes and test them, but this can be very expensive and time-consuming. Even with an existing system, it is likely to be impossible or impractical to experiment with the actual system. For example, it is not feasible to study economic systems by arbitrarily changing the supply and demand of goods. Consequently, system studies are generally conducted with a model of the system. For the purpose of most studies, it is not necessary to consider all the details of a system; so a model is not only a substitute for a system, it is also a simplification of the system, (14).

We define a *model* as the body of information about a system gathered for the purpose of studying the system.[2] Since the purpose of the study will determine the nature of the information that is gathered, there is no unique model of a system. Different models of the same system will be produced by different analysts interested in different aspects of the system or by the same analyst as his understanding of the system changes.

The task of deriving a model of a system may be divided broadly into two subtasks: establishing the model structure and supplying the data. Establishing the structure determines the system boundary and identifies the entities, attributes, and

[2] In the case of a physical model, the information is embodied in the properties of the model, in contrast to the symbolic representation in a mathematical model.

activities of the system. The data provide the values the attributes can have and define the relationships involved in the activities. The two jobs of creating a structure and providing the data are defined as parts of one task rather than as two separate tasks, because they are usually so intimately related that neither can be done without the other. Assumptions about the system direct the gathering of data, and analysis of the data confirms or refutes the assumptions. Quite often, the data gathered will disclose an unsuspected relationship that changes the model structure.

To illustrate this process, consider the following description of a supermarket, (3).

Shoppers needing *several items* of shopping *arrive* at a supermarket. They *get* a *basket*, if one is *available*, carry out their *shopping*, and then *queue* to *check-out* at one of the *several counters*.

After checking-out, they *return* the *basket* and *leave*.

Certain words have been italicized because they are considered to be key words that point out some feature of the system that must be reflected in the model. Essentially the same description is rewritten in Fig. 1-4 to identify the entities,

ENTITY	ATTRIBUTE	ACTIVITY
SHOPPER	NO OF ITEMS	ARRIVE
		GET
BASKET	AVAILABILITY	
		SHOP
		QUEUE
		CHECK-OUT
COUNTER	NUMBER OCCUPANCY	
		RETURN
		LEAVE

Figure 1-4. Elements of a supermarket model.

attributes, and activities. Notice that the concept of a supermarket as a whole does not appear as an entity. It defines the system boundary and therefore distinguishes between the system and its environment. The arrival of customers in this description of the system will be regarded as an exogenous activity affecting the system from the environment. If, in contrast, the study objectives include analyzing the effects of car parking facilities on supermarket business, the boundary of the system would need to include the parking lot. The arrival of a customer in the supermarket depends upon finding a parking space, which can depend upon the departure of customers. Customer arrivals in the supermarket then become an endogenous activity; the arrival of cars becomes an exogenous activity.

Other decisions about the system study objectives are implied in the model. The number of items of shopping is represented as an attribute of the shopper, but no distinction has been made about the type of item. Secondly, no provision has been made in the system model for the effects of congestion on shopping time. If these decisions are not in keeping with the study objectives, another form of model must be used. In the first case, where type of item is to be distinguished, it is necessary to define several attributes for each customer, one for each type of item to be purchased. In the second case, where allowance for congestion must be made, two approaches could be taken. It may be necessary to introduce new entities representing the various sections of the supermarket and establish as attributes the number of customers they can serve simultaneously. Alternatively, the activity of shopping could be represented by a function in which shopping time depends upon the number of shoppers in the supermarket.

It is not suggested that Fig. 1-4 represents a formal process by which a transition can be made from a verbal description of a system to the structure of a model. It merely illustrates the process involved in forming a model.

1-6
Types of Models

Models used in system studies have been classified in many ways, (8), (9), (16), and (17). The classifications that will be used here are illustrated in Fig. 1-5. Models will first be separated into *physical models* or *mathematical models*.

Physical models are based on some analogy between such systems as mechanical and electrical, (4) and (11), or electrical and hydraulic, (10). In a physical model of a system, the system attributes are represented by such measurements as a voltage or the position of a shaft. The system activities are reflected in the physical laws that drive the model. For example, the rate at which the shaft of a direct current motor turns depends upon the voltage applied to the motor. If the applied voltage is used to represent the velocity of a vehicle, then the number of revolutions of the shaft is a measure of the distance the vehicle has traveled; the higher the voltage, or velocity, the greater is the buildup of revolutions, or distance covered, in a given time.

Mathematical models, of course, use symbolic notation and mathematical equations to represent a system. The system attributes are represented by variables,

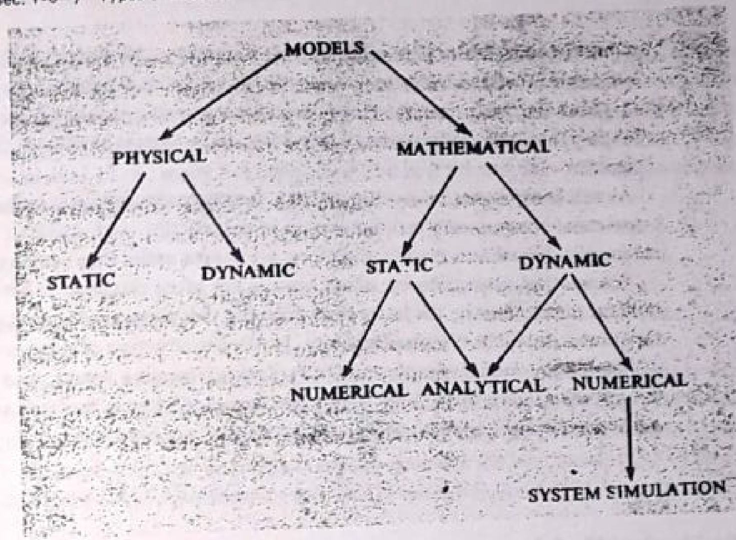


Figure 1-5. Types of models.

and the activities are represented by mathematical functions that interrelate the variables.

A second distinction will be between *static models* and *dynamic models*. Static models can only show the values that system attributes take when the system is in balance. Dynamic models, on the other hand, follow the changes over time that result from the system activities.

In the case of mathematical models, a third distinction is the technique by which the model is "solved," that is, actual values are assigned to system attributes. A distinction is made between *analytical* and *numerical* methods. Applying analytical techniques means using the deductive reasoning of mathematical theory to solve a model. In practice, only certain forms of equations can be solved. Using analytical techniques, therefore, is a matter of finding the model that can be solved and best fits the system being studied. For example, linear differential equations can be solved. Knowing this, an engineer who restricts the description of a system to that form will derive a model that can be solved analytically.

Numerical methods involve applying computational procedures to solve equations. To be strictly accurate, any assignment of numerical values that uses mathematical tables involves numerical methods, since tables are derived numerically. The distinction being drawn here is that analytical methods produce solutions in tractable form, meaning a form where values can be assigned from available tables. Making use of an analytical solution may, in fact, require a considerable amount of computation. For example, the solution may be derived in the form of a complicated integral which then needs to be expanded as a power series for evaluation. However, mathematical theory for making such expansions exists, and, in principle, any degree of accuracy in the solution is obtainable if sufficient effort is expended.

As will be discussed more fully in Chap. 3, system simulation is considered to be a numerical computation technique used in conjunction with dynamic mathematical models. Simulation models, therefore, are shown under that heading in Fig. 1-5.

Yet another distinction by which models are often classified is between deterministic and stochastic: the latter term meaning that there are random processes in the system. As will be discussed in Chap. 14, the introduction of stochastic processes in a simulation model complicates the task of interpreting results, and it increases the amount of work to be done. It does not, however, change the basic technique by which simulation is applied, so this distinction has not been made in Fig. 1-5.

1-7 Static Physical Models

The best known examples of physical models are scale models. In shipbuilding, making a scale model provides a simple way of determining the exact measurements of the plates covering the hull, rather than having to produce drawings of complicated, three-dimensional shapes. Scientists have used models in which spheres represent atoms, and rods or specially shaped sheets of metal connect the spheres to represent atomic bonds. A model of this nature played an important role in the deciphering of the DNA molecule, work that was the subject of a Nobel Prize award, (19). These models are static physical models. They are sometimes said to be *iconic* models, a term meaning "look-alike" (7).

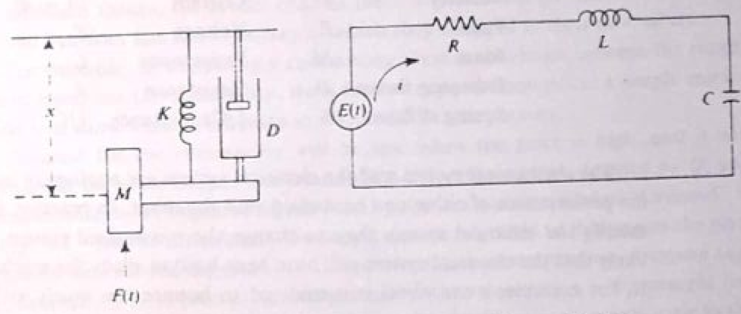
Scale models are also used in wind tunnels and water tanks (1) in the course of designing aircraft and ships. Although air is blown over the model, or the model is pulled through the water, these are static physical models because the measurements that are taken represent attributes of the system being studied under one set of equilibrium conditions. In this case, the measurements do not translate directly into system attribute values. Well known laws of similitude are used to convert measurements on the scale model to the values that would occur in the real system, (5) and (13).

Sometimes, a static physical model is used as a means of solving equations with particular boundary conditions. There are many examples in the field of mathematical physics where the same equations apply to different physical phenomena. For example, the flow of heat and the distribution of electric charge through space

can be related by common equations. In general, these equations can only be solved for simple-shaped bodies. In practice, solutions are needed for specific, complicated shapes. The distribution of heat in a body can be predicted by enclosing a space that has the same shape as the body, and measuring the charge in the space when the surface of the space has been electrified in a manner that reflects the way heat will be injected into the body, (2).

1-8 Dynamic Physical Models

Dynamic physical models rely upon an analogy between the system being studied and some other system of a different nature, the analogy usually depending upon an underlying similarity in the forces governing the behavior of the systems. To illustrate this type of physical model, consider the two systems shown in Fig. 1-6. Figure 1-6(a) represents a mass that is subject to an applied force $F(t)$ varying



(a) Mechanical System (b) Electrical System

Figure 1-6. Analogy between mechanical and electrical systems.

with time, a spring whose force is proportional to its extension or contraction, and a shock absorber that exerts a damping force proportional to the velocity of the mass. The system might, for example, represent the suspension of an automobile wheel when the automobile body is assumed to be immobile in a vertical direction. It can be shown that the motion of the system is described by the following differential equation:[3]

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

[3]The derivation of this equation will be given in Sec. 4-2.

where x is the distance moved,
M is the mass,
K is the stiffness of the spring,[4]
D is the damping factor of the shock absorber.

Figure 1-6(b) represents an electrical circuit with an inductance L, a resistance R, and a capacitance C, connected in series with a voltage source that varies in time according to the function $E(t)$. If q is the charge on the capacitance, it can be shown that the behavior of the circuit is governed by the following differential equation:

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = \frac{E(t)}{C}$$

Inspection of these two equations shows that they have exactly the same form and that the following equivalences occur between the quantities in the two systems:

Displacement	x	Charge	q
Velocity	$\dot{x}$	Current	$I\,(=\dot{q})$
Force	F	Voltage	E
Mass	M	Inductance	L
Damping factor	D	Resistance	R
Spring stiffness	K	1/Capacitance	$1/C$

The mechanical system and the electrical system are analogs of each other, and the performance of either can be studied with the other. In practice, it is simpler to modify the electrical system than to change the mechanical system, so it is more likely that the electrical system will have been built to study the mechanical system. If, for example, a car wheel is considered to bounce too much with a particular suspension system, the electrical model will demonstrate this fact by showing that the charge (and, therefore, the voltage) on the condenser oscillates excessively. To predict what effect a change in the shock absorber or spring will have on the performance of the car, it is only necessary to change the values of the resistance or

[4]The constant K is included on the right-hand side of the equation as a matter of convenience. To demonstrate the characteristic behavior of a system, it is customary to show its response to a step-function of force, that is, a steady force applied when the body is stationary. The step-function is usually taken to be of unit magnitude.

If the system is stable, its response settles to a steady value. With K on the right-hand side of the equation, the steady value is always 1. This makes it possible to draw the responses for different values of coefficients on a common graph, as will be done in Fig. 1-9. Interpreting $F(t)$ as being a unit step-function means that the applied force is actually a step-function of magnitude K.

condenser in the electrical circuit and observe the effect on the way the voltage varies.

If, in fact, the mechanical system were as simple as illustrated, it could be studied by solving the mathematical equation derived in establishing the analogy. However, effects can easily be introduced that would make the mathematical equation difficult to solve. For example, if the motion of the wheel is limited by physical stops, a non-linear equation that is difficult to solve will be needed to describe the system. It is easy to model the effect electrically by placing limits on the voltage that can exist on the capacitance.

1-9
Static Mathematical Models

A static model gives the relationships between the system attributes when the system is in equilibrium. If the point of equilibrium is changed by altering any of the attribute values, the model enables the new values for all the attributes to be derived but does not show the way in which they changed to their new values.

For example, in marketing a commodity there is a balance between the supply and demand for the commodity. Both factors depend upon price: a simple *market model* will show what is the price at which the balance occurs.

Demand for the commodity will be low when the price is high, and it will increase as the price drops. The relationship between demand, denoted by Q, and price, denoted by P, might be represented by the straight line marked "Demand" in Fig. 1-7.[5] On the other hand, the supply can be expected to increase as the price increases, because the suppliers see an opportunity for more revenue. Suppose supply, denoted by S, is plotted against price, and the relationship is the straight line marked "Supply" in Fig. 1-7. If conditions remain stable, the price will settle to the point at which the two lines cross, because that is where the supply equals the demand.

Since the relationships have been assumed linear, the complete market model can be written mathematically as follows:

$$Q = a - bP$$
$$S = c + dP$$
$$S = Q$$

[5]This description makes price the independent variable, and demand the dependent variable. However, Figs. 1-7 and 1-8 place price along the vertical axis in order to conform with the practice normally used in the literature of economics. See (18).

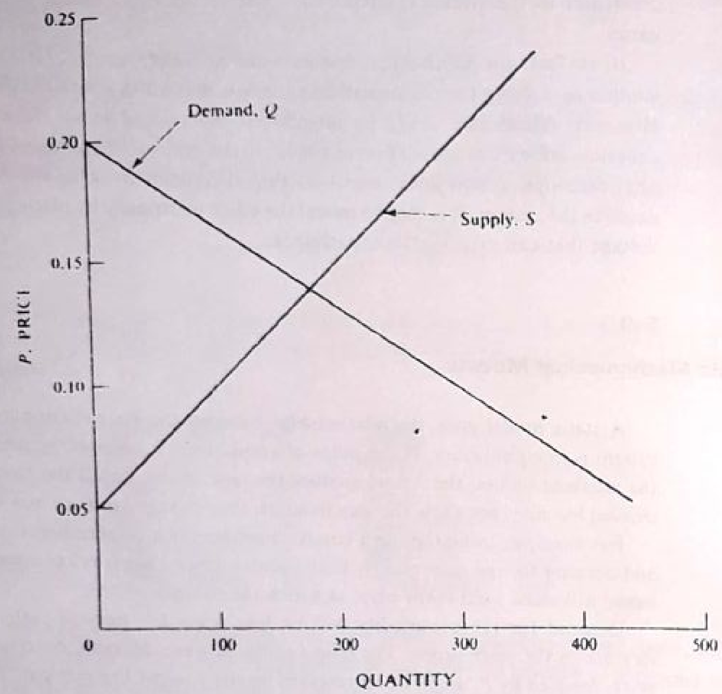


Figure 1-7. Linear market model.

The last equation states the condition for the market to be cleared: it says supply equals demand and, so, determines the price to which the market will settle.

For the model to correspond to normal market conditions in which demand goes down and supply increases as price goes up the coefficients b and d need to be positive numbers. For realistic, positive results, the coefficient a must also be positive. Figure 1-7 has been plotted for the following values of the coefficients:

$$a = 600$$
$$b = 3{,}000$$
$$c = -100$$
$$d = 2{,}000$$

The fact that linear relationships have been assumed allows the model to be

solved analytically. The equilibrium market price, in fact, is given by the following expression:

$$P = \frac{a - c}{b - d}$$

With the chosen values, the equilibrium price is 0.14, which corresponds to a supply of 180.

More usually, the demand will be represented by a *curve* that slopes downwards, and the supply by a curve that slopes upwards, as illustrated in Fig. 1-8. It may not

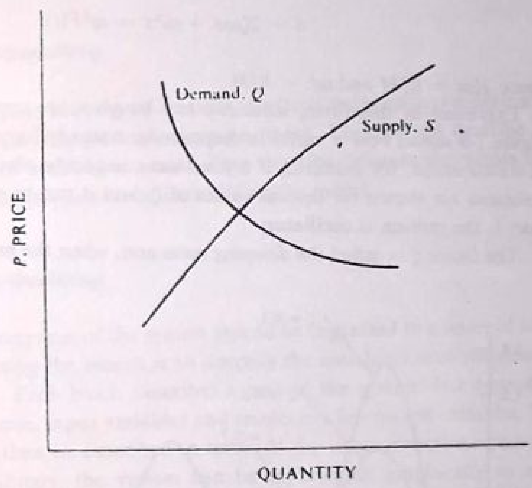


Figure 1-8. Non-linear market model.

then be possible to express the relationships by equations that can be solved. Some numeric method is then needed to solve the equations. Drawing the curves to scale and determining graphically where they intersect is one such method.

In practice, it is difficult to get precise values for the coefficients of the model. Observations over an extended period of time, however, will establish the slopes (that is, the values of b and d) in the neighborhood of the equilibrium point, and, of course, actual experience will have established equilibrium prices under various conditions. The values depend upon economic factors, so the observations will usually attempt to correlate the values with the economy, allowing the model to be used as a means of forecasting changes in market conditions for anticipated economic changes.

1-10

Dynamic Mathematical Models

A dynamic mathematical model allows the changes of system attributes to be derived as a function of time. The derivation may be made with an analytical solution or with a numerical computation, depending upon the complexity of the model. The equation that was derived to describe the behavior of a car wheel is an example of a dynamic mathematical model; in this case, an equation that can be solved analytically. It is customary to write the equation in the form

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = \omega^2 F(t) \qquad (1\text{-}1)$$

where $2\zeta\omega = D/M$ and $\omega^2 = K/M$.

Expressed in this form, solutions can be given in terms of the variable ωt. Figure 1-9 shows how x varies in response to a steady force applied at time $t = 0$ as would occur, for instance, if a load were suddenly placed on the automobile. Solutions are shown for several values of ζ, and it can be seen that when ζ is less than 1, the motion is oscillatory.

The factor ζ is called the *damping ratio* and, when the motion is oscillatory, the

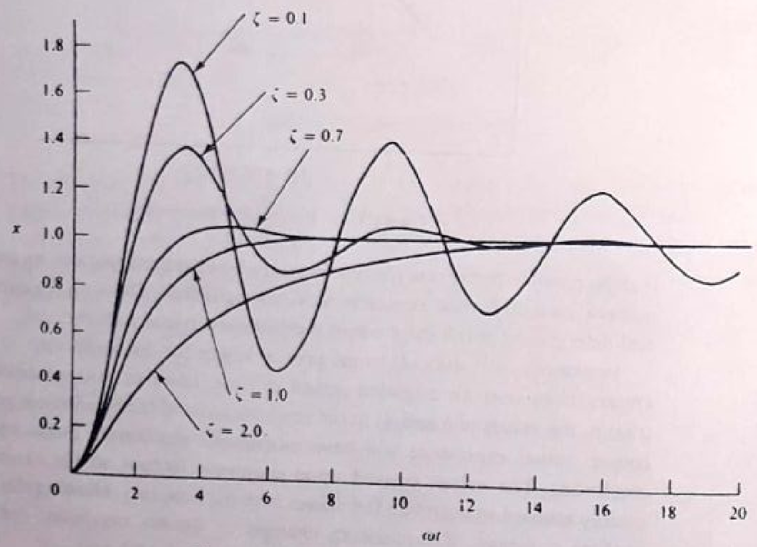


Figure 1-9. Solutions of second-order equations.

frequency of oscillation is determined from the formula

$$\omega = 2\pi f$$

where f is the number of cycles per second.

Suppose a case is selected as representing a satisfactory frequency and damping. The relationships given above between ζ, ω, M, K, and D show how to select the spring and shock absorber to get that type of motion. For example, the condition for the motion to occur without oscillation requires that $\zeta \geq 1$. It can be deduced from the definition of ζ and ω that the condition requires that $D^2 \geq 4MK$.

1-11

Principles Used in Modeling

It is not possible to provide rules by which mathematical models are built, but a number of guiding principles can be stated. They do not describe distinct steps carried out in building a model. They describe different viewpoints from which to judge the information to be included in the model.

(a) Block-building

The description of the system should be organized in a series of *blocks*. The aim in constructing the blocks is to simplify the specification of the interactions within the system. Each block describes a part of the system that depends upon a few, preferably one, input variables and results in a few output variables. The system as a whole can then be described in terms of the interconnections between the blocks. Correspondingly, the system can be represented graphically as a simple block diagram.

The description of a factory given in Fig. 1-2 is a typical example of a block diagram. Each department of the factory has been treated as a separate block, with the inputs and outputs being the work passed from department to department. The fact that the departments might occupy the same floor space and might use the same personnel or the same machines has been ignored.

(b) Relevance

The model should only include those aspects of the system that are relevant to the study objectives. As an example, if the factory system study aims to compare the effects of different operating rules on efficiency, it is not relevant to consider the hiring of employees as an activity. While irrelevant information in the model may not do any harm, it should be excluded because it increases the complexity of the model and causes more work in solving the model.

(c) Accuracy

The accuracy of the information gathered for the model should be considered. In the aircraft system, for example, the accuracy with which the movement of the aircraft is described depends upon the representation of the airframe. It may suffice to regard the airframe as a rigid body and derive a very simple relationship between control surface movement and aircraft heading, or it may be necessary to recognize the flexibility of the airframe and make allowance for vibrations in the structure. An engineer responsible for estimating the fuel consumption may be satisfied with the simple representation. Another engineer, responsible for considering the comfort of the passengers, needs to consider vibrations and will want the detailed description of the airframe.

(d) Aggregation

A further factor to be considered is the extent to which the number of individual entities can be grouped together into larger entities. The general manager of the factory may be satisfied with the description that has been given. The production control manager, however, will want to consider the shops of the departments as individual entities.

In some studies, it may be necessary to construct artificial entities through the process of aggregation. For example, an economic or social study will usually treat a population as a number of social classes and conduct a study as though each social class were a distinct entity.

Similar considerations of aggregation should be given to the representation of activities. For example, in studying a missile defense system, it may not be necessary to include the details of computing a missile trajectory for each firing. It may be sufficient to represent the outcome of many firings by a probability function.

Exercises

1-1 Extract from the following description the entities, attributes, and activities of the system. Ships arrive at a port. They dock at a berth if one is available; otherwise, they wait until one becomes available. They are unloaded by one of several work gangs whose size depends upon the ship's tonnage. A warehouse contains a new cargo for the ship. The ship is loaded and then departs. Suggest two exogenous events (other than arrivals) that may need to be taken into account.

1-2 Name three or four of the principal entities, attributes, and activities to be considered if you were to simulate the operation of (a) a gasoline filling station, (b) a cafeteria, (c) a barber shop.

1-3 A new bus route is to be added to a city, and the traffic manager is to determine how many extra buses will be needed. What are the three key attributes of the passengers and buses that he should consider? If the company manager wants to assess the effect of the new route on the transit system as a whole, how would you suggest he aggregate the features of the new line to form part of a total system model? Would you suggest a continuous or discrete model for the traffic manager and the general manager?

1-4 In the automobile wheel suspension system, it is found that the shock absorber damping force is not strictly proportional to the velocity of the wheel. There is an additional force component equal to D_2 times the acceleration of the wheel. Find the new conditions for ensuring that the wheel does not oscillate.

1-5 A woman does her shopping on Mondays, Wednesdays, and Fridays. If it is fine, she walks to the stores; otherwise, she takes a bus. She always takes the bus home. On Tuesdays, she visits her daughter, traveling there and back by bus. Assuming that information is available about the day of the week and the state of the weather, draw a flow chart of her movements.

1-6 In the aircraft system, suppose the control surface angle y is made to be A times the error signal. The response of the aircraft to the control surface is found to be $I\ddot{\theta}_o + D\dot{\theta}_o = Ky$. Find the conditions under which the aircraft motion is oscillatory.

1-7 Suppose the automobile body in the suspension system example is not stationary. Consider the body to have a mass of M_1, and assume that its motion is determined by the force of gravity and the reaction with the suspension system. Construct a model for the motions of the wheel and body.

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