APLIKASI TRANFORMASI LAPLACE PADA ANALISA RANGKAIAN LISTRIK

TABLE 19.1 Short Table of Laplace Transforms

$f(t)$	$F(s)$
1. $\delta(t)$	1
2. $u(t)$	$\frac{1}{s}$
3. e^{-at}	$\frac{1}{s+a}$
4. $\sin kt$	$\frac{k}{s^2+k^2}$
5. $\cos kt$	$\frac{s}{s^2+k^2}$
6. $e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
7. $e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
8. t	$\frac{1}{s^2}$
9. te^{-at}	$\frac{1}{(s+a)^2}$
10. $\frac{t^{n-1}e^{-at}}{(n-1)!}$	$\frac{1}{(s+a)^n}$; $n = 1, 2, 3, \ldots$

Contoh Inverse Transformasi Laplace

Contoh laplace transform

1. $F(s) = \frac{6}{s+4} + \frac{2}{s^2+9} - \frac{3}{s}$ *

$f(t) = 6\,\mathcal{L}^{-1}\left[\frac{1}{s+4}\right] + \frac{2}{3}\,\mathcal{L}^{-1}\left[\frac{3}{s^2+3^2}\right] - 3\,\mathcal{L}^{-1}\left[\frac{1}{s}\right]$

$f(t) = 6 \cdot e^{-4t} + \frac{2}{3}\sin 3t - 3$

2. $F(s) = \frac{2(s+10)}{(s+1)(s+4)}$

$F(s) = \frac{2(s+10)}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4} \Rightarrow A = 6, \; B = -4$

$F(s) = \frac{6}{s+1} - \frac{4}{s+4} = 6 \cdot e^{-t} - 4 \cdot e^{-4t}$

3. $F(s) = \frac{2s}{(s+1)(s+2)(s+3)}$

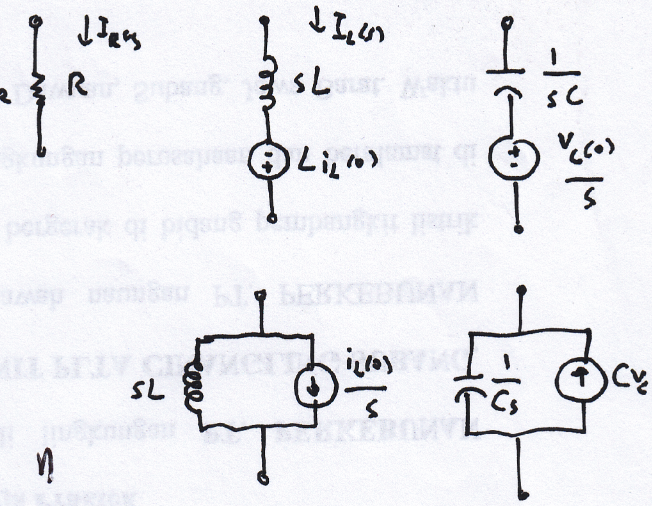
$= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$ $\quad A = -1, \; B = 4, \; C = -3$

$f(t) = -e^{-t} + 4e^{-2t} - 3e^{-3t}$

TRANFORMASI KOMPONEN

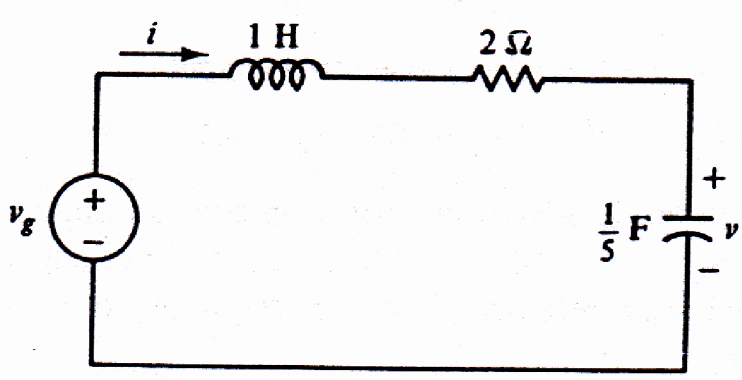
Laplace Transform

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin kt$	$\frac{k}{s^2+k^2}$
$\cos kt$	$\frac{s}{s^2+k^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
t	$\frac{1}{s^2}$
te^{-at}	$\frac{1}{(s+a)^2}$



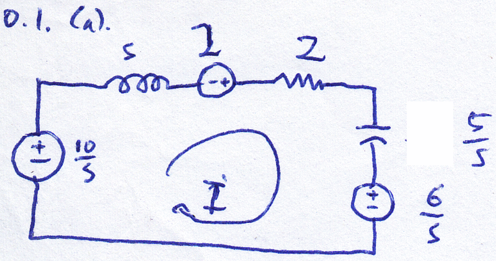
CONTOH-CONTOH ANALISA SOAL

20.1 Use the describing equation and Laplace transforms to find i for $t > 0$ if $v(0) = 6$ V, $i(0) = 2$ A, and (a) $v_g = 10$ V and (b) $v_g = 4\cos t$ V.



PROBLEM 20.1

20.1. (a).



$$-\frac{10}{s} + sI - 2 + 2I + \frac{5}{s}I + \frac{6}{s} = 0$$

$$I\left(s+2+\frac{5}{s}\right) = \frac{10}{s} + 2 - \frac{6}{s}$$

$$I = \frac{\frac{4}{s}+2}{s+2+\frac{5}{s}} \times \frac{s}{s} = \frac{4+2s}{s^2+2s+5}$$

$$s^2+2s+5 = (s+1)^2+4$$

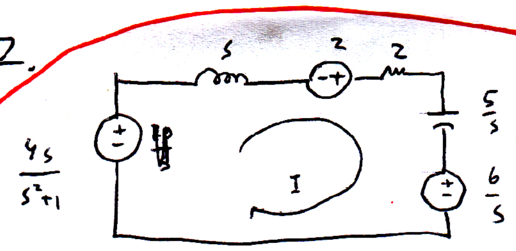
$$I = \frac{4}{(s+1)^2+4} + \frac{2s}{(s+1)^2+4}$$

$$= 2\cdot\frac{2}{(s+1)^2+2^2} + 2\frac{(s+1)}{(s+1)^2+2^2} - \frac{2}{(s+1)^2+2^2}$$

$$= 2\frac{(s+1)}{(s+1)^2+2^2} + \frac{2}{(s+1)^2+2^2}$$

$$= 2\cdot e^{-t}\cos 2t + e^{-t}\,\cancel{\cos}\,\sin 2t$$

2.



$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$

$$-\frac{4s}{s^2+1} + Is - 2 + 2I + \frac{5}{s}I + \frac{6}{s} = 0$$

$$I\left(s+2+\frac{5}{s}\right) = \frac{4s}{s^2+1} - \frac{6}{s} + 2$$

$$I = \frac{\frac{4s}{s^2+1} - \frac{6}{s} + 2}{\left(s+2+\frac{5}{s}\right)} \times \frac{s}{s}$$

$$= \frac{\frac{4s^2}{s^2+1} - 6 + 2s}{s^2+2s+5}$$

$$I = \frac{4s^2 + (2s-6)(s^2+1)}{(s^2+1)(s^2+2s+5)} = \frac{4s^2}{(s^2+1)(s^2+2s+5)} + \frac{(2s-6)\cancel{(s^2+1)}}{\cancel{(s^2+1)}(s^2+2s+5)}$$

$$\frac{2s-6}{s^2+2s+5} = \frac{2s}{(s+1)^2+2^2} - \frac{6}{(s+1)^2+2^2}$$

$$= 2\frac{(s+1)}{(s+1)^2+2^2} - \frac{2}{(s+1)^2+2^2} - 3\frac{2}{(s+1)^2+2^2}$$

$$= 2\frac{(s+1)}{(s+1)^2+2^2} - 4\cdot\frac{2}{(s+1)^2+2^2}$$

$$i_2(t) = 2\cdot e^{-t}\cos 2t - 4\cdot e^{-t}\sin 2t = \frac{10}{5}e^{-t}\cos 2t - \frac{20}{5}e^{-t}\sin 2t$$

$$\frac{4s^2}{(s^2+1)(s^2+2s+5)} = \frac{A}{s^2+1} + \frac{Bs}{s^2+1} + \frac{D\cdot 2}{(s+1)^2+2^2} + \frac{E\cdot(s+1)}{(s+1)^2+2^2}$$

$$4s^2 = A(s^2+2s+5) + Bs(s^2+2s+5) + 2D(s^2+1) + E(s+1)(s^2+1)$$
$$= As^2 + 2As + 5A + Bs^3 + 2Bs^2 + 5Bs + 2Ds^2 + 2D + Es^3 + Es^2 + Es + E$$

$5A + 2D + E = 0$

$2As + 5Bs + Es = 0$

$As^2 + 2Bs^2 + 2Ds^2 + Es^2 = 4$

$Bs^3 + Es^3 = 0$

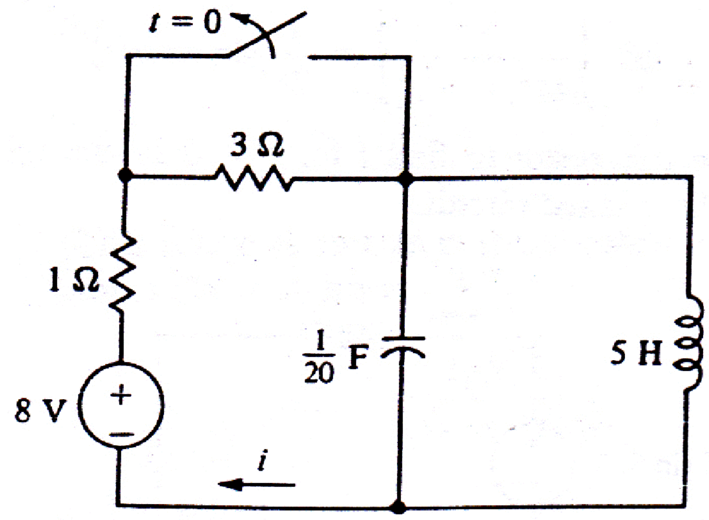
$A = -\frac{8}{10} = -\frac{4}{5}$ $\quad D = \frac{11}{5}$

$B = \frac{2}{5}$ $\quad E = -\frac{2}{5}$

$$i_1 = -\frac{4}{5}\sin 2t + \frac{2}{5}\cos 2t + \frac{11}{5}(e^{-t}\cdot\sin 2t) - \frac{2}{5}(e^{-t}\cdot\cos 2t)$$

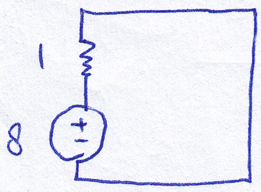
$$i = -\frac{4}{5}\sin 2t + \frac{2}{5}\cos 2t + \frac{8}{5}e^{-t}\cos 2t - \frac{9}{5}e^{-t}\cdot\sin 2t$$

20.3 Use the describing equation and Laplace transforms to find i for $t > 0$ if the circuit is in steady state at $t = 0^-$.



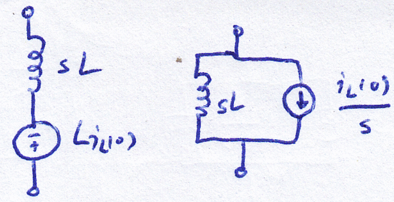
PROBLEM 20.3

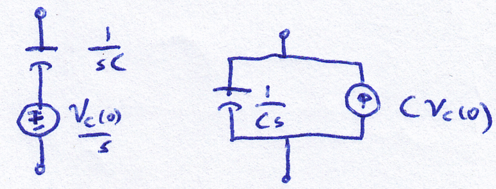
20.3. $t=0^-$



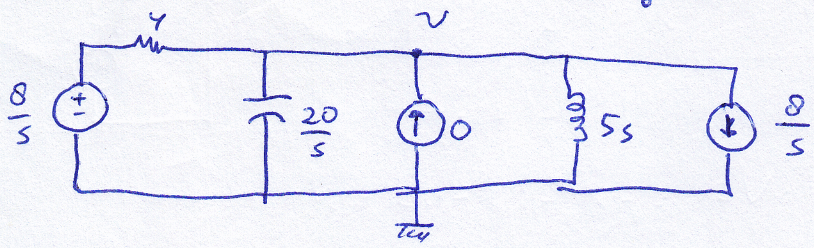
$V_c = 0$

$i_L = 8A$





$t = 0^+$



Node v:

$$\frac{V-\frac{8}{s}}{4} + \frac{V\cdot s}{20} + \frac{V}{5s} + \frac{8}{s} = 0 \qquad \times 20s$$

$$V\cdot 5\cdot s - 40 + Vs^2 + 4V + 160 = 0$$

$$V(s^2+5s+4) = \cancel{200} \ -120$$

$$V = \frac{\cancel{200}\ -120}{(s+4)(s+1)} = \frac{A}{s+4} + \frac{B}{s+1}$$

$$-120 = As + A + Bs + 4B$$

$$A + B = 0$$

$$A + 4B = \cancel{20}\ -120$$

$$-3B = 120$$

$$B = -40$$

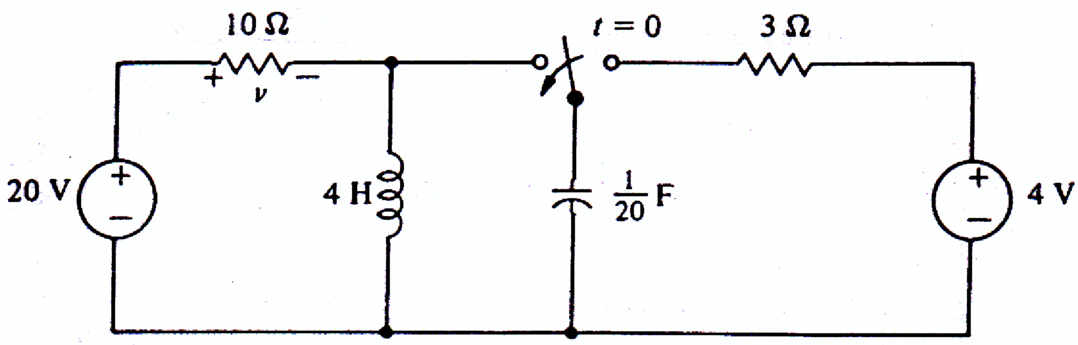
$$A = 40$$

$$V = \frac{40}{s+4} - \frac{40}{s+1}$$

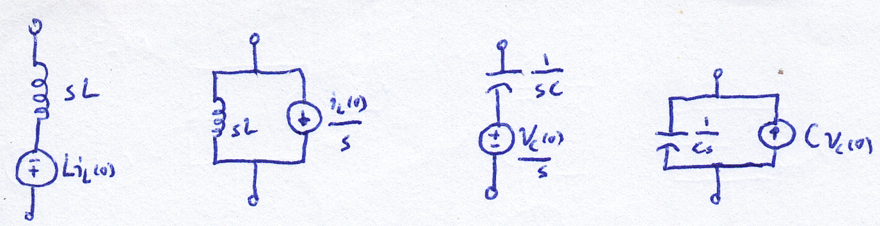
$$V(t) = 40\cdot e^{-4t} - 40\cdot e^{-t} \text{ Volt}$$

$$i = \frac{8-V_c}{4} = 2 - 10\cdot e^{-4t} + 10\cdot e^{-t} \text{ A}$$

9.29 Find v for $t > 0$ if the circuit is in steady state at $t = 0^-$.

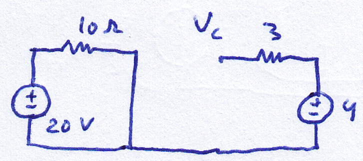


PROBLEM 9.20



20.9

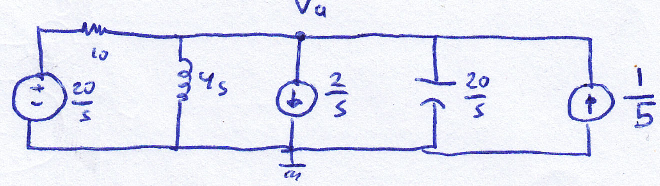
$t = 0^-$



$i_L(0) = \frac{20}{10} = 2\ A$

$V_c = 4V$

$t = 0^+$



$$\frac{V_a - \frac{20}{s}}{10} + \frac{V_a}{4s} + \frac{2}{s} + \frac{V_a \cdot s}{20} - \frac{1}{5} = 0 \qquad \times 20s$$

$$2V_a \cdot s - 40 + 5V_a + 40 + V_a s^2 - 4s = 0$$

$$V_a\left[s^2 + 2s + 5\right] = 4s$$

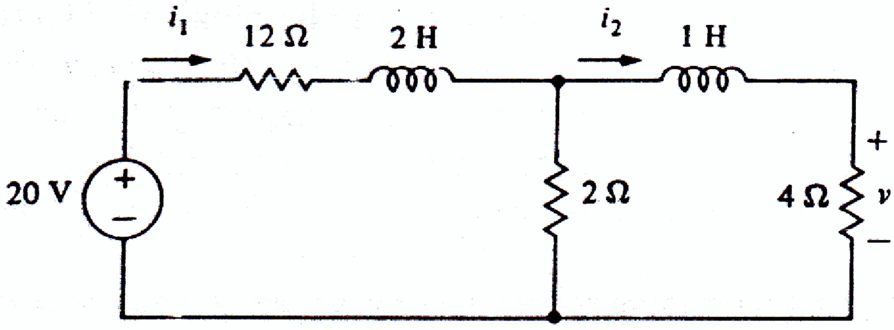
$$V_a = \frac{4s}{s^2 + 2s + 5} = \frac{4s}{(s+1)^2 + 2^2}$$

$$= 4 \cdot \frac{s+1}{(s+1)^2 + 2^2} - 2 \cdot \frac{2}{(s+1)^2 + 2^2}$$

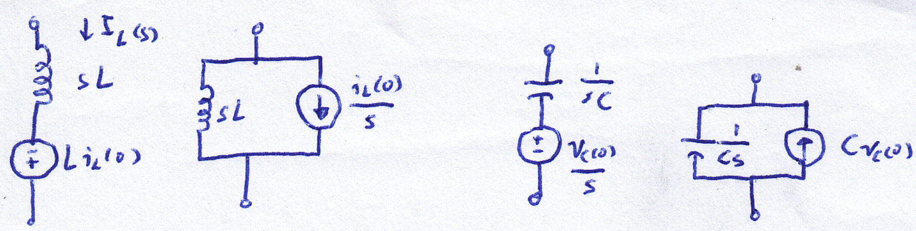
$$V_a(t) = 4 \cdot e^{-t} \cdot \cos 2t - 2 \cdot e^{-t} \cdot \sin 2t$$

$$V = 20 - 4 \cdot e^{-t} \cdot \cos 2t + 2 \cdot e^{-t} \cdot \sin 2t$$

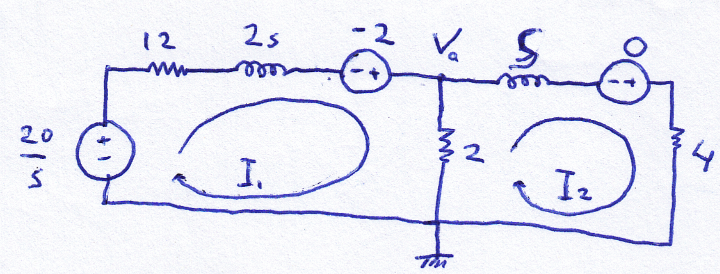
20.19 Use the transformed circuit to find v for $t > 0$ if $i_1(0) = -1$ A and $i_2(0) = 0$.



PROBLEM 20.19



20. 19



Loop I_1:

$$-\frac{20}{s} + 12I_1 + 2sI_1 - 2 + 2I_1 - 2I_2 = 0 \quad \times s$$

$$-20 + 12I_1 s + 2I_1 s^2 + 2s + 2I_1 s - 2I_2 s = 0$$

$$I_1(2s^2 + 14s) - 2I_2 s = 20 \mp 2s$$

$$I_1(s^2 + 7s) - I_2 s = (10 \mp s)$$

Loop I_2:

$$2I_2 - 2I_1 + sI_2 + 4I_2 = 0$$

$$-2I_1 + (6+s)I_2 = 0$$

$$2I_1 = (6+s)I_2$$

$$I_1 = \frac{(6+s)}{2} I_2$$

$$2 I_1(s^2+7s) - 2I_2 s = 20 \mp 2s$$

$$(6+s)I_2(s^2+7s) - 2I_2 s = 20 \mp 2s$$

$$(s^3+7s^2+6s^2+42s)I_2 - 2I_2 s = 20 \mp 2s$$

$$\cancel{I_2(s^3+11s^2}$$

$$I_2(s^3+13s^2+40s) = 20 \mp 2s$$

$$I_2 = \frac{20 \mp 2s}{s(s^2+13s+40)}$$

$$= \frac{20 \mp 2s}{s(s+8)(s+5)} = \frac{A}{s} + \frac{B}{s+8} + \frac{C}{s+5}$$

$$20 \mp 2s = A(s+8)(s+5) + Bs(s+5) + C(s+8)s$$

$$= As^2 + A13s + 40A + Bs^2 + B5s + Cs^2 + C8s$$

$$0 = A + B + C \qquad 20 = 40A$$

$$-2 = 13A + 5B + 8C \qquad A = \frac{1}{2}$$

$$B + C = -\frac{1}{2} \quad \Big| \times 5$$

$$\cancel{A = \frac{1}{2}} \quad 5B + 8C = -8{,}5$$

$$5B + 5C = -2{,}5$$

$$5B + 8C = -8{,}5 \quad -$$

$$-3C = 6$$

$$C = -2$$

$$B = \cancel{-2{,}5} \quad 1{,}5$$

$$I_2 = \frac{1}{2s} + \frac{\cancel{2{,}5}\ 1{,}5}{s+8} \mp \frac{2}{s+5}$$

$$V = \frac{2}{s} + \frac{6}{s+8} - \frac{8}{s+5}$$

$$V = 2 + 6 \cdot e^{-8t} - 8 \cdot e^{-5t}$$