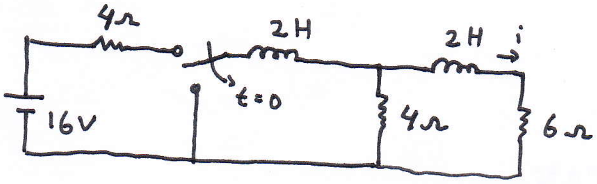
Find $i$ for $t>0$ if the circuit is in steady state at $t=0^-$

9.3




Answer :

Circuit at $t=0^-$


Loop $i_{L_1}$:

$$-16+4i_{L_1}+4(i_{L_1}-i_{L_2})=0$$

$$8i_{L_1}-4i_{L_2}=16 \quad \ldots (1)$$

Loop $i_{L_2}$:

$$4(i_{L_2}-i_{L_1})+6i_{L_2}=0$$

$$-4i_{L_1}+10i_{L_2}=0 \quad \ldots (2)$$

(1) & (2):

$$\begin{aligned} 8i_{L_1}-4i_{L_2}&=16 \quad &\times 1\\ -4i_{L_1}+10i_{L_2}&=0 \quad &\times 2 \end{aligned}$$

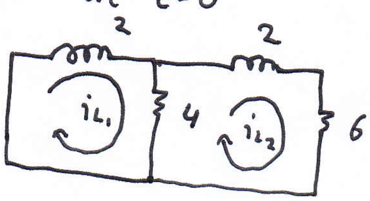
$$\begin{aligned} 8i_{L_1}-4i_{L_2}&=16\\ -8i_{L_1}+20i_{L_2}&=0 \quad + \end{aligned}$$

$$16i_{L_2}=16$$

$$i_{L_2}=1$$

$$i_{L_1}=\frac{10}{4}=2,5$$

Circuit at $t=0^+$




Loop $i_{L_1}$

$$2\frac{di_{L_1}}{dt}+4(i_{L_1}-i_{L_2})=0$$

$$2\frac{di_{L_1}}{dt}+4i_{L_1}-4i_{L_2}=0 \quad \ldots (3)$$

Loop $i_{L_2}$:

$$4(i_{L_2}-i_{L_1})+2\frac{di_{L_2}}{dt}+6i_{L_2}=0$$

$$4i_{L_1}=2\frac{di_{L_2}}{dt}+10i_{L_2} \quad \ldots (4)$$

differential:

$$4\frac{di_{L_1}}{dt}=2\frac{d^2i_{L_2}}{dt^2}+10\frac{di_{L_2}}{dt} \quad \times\frac{1}{2}$$

$$2\frac{di_{L_1}}{dt}=\frac{d^2i_{L_2}}{dt^2}+5\frac{di_{L_2}}{dt} \quad \ldots (5)$$

(3), (4), (5):

$$\left(\frac{d^2i_{L_2}}{dt^2}+5\frac{di_{L_2}}{dt}\right)+\left(2\frac{di_{L_2}}{dt}+10i_{L_2}\right)-4i_{L_2}=0$$

$$\frac{d^2i_{L_2}}{dt^2}+7\frac{di_{L_2}}{dt}+6i_{L_2}=0$$

Transformasi :

$$s^2+7s+6=0$$

$$(s+6)(s+1)=0$$

$$s_1=-1 \qquad s_2=-6$$

$$i_{L_2}=A_1\cdot e^{-t}+A_2\cdot e^{-6t}$$

dari pers (*):

$$i_{L_1} = \frac{1}{2}\frac{di_{L_2}}{dt} + \frac{5}{2} i_{L_2}$$

$$= \frac{1}{2}\left(-A_1 e^{-t} - 6A_2 e^{-6t}\right) + \frac{5}{2}\left(A_1 \cdot e^{-t} + A_2 \cdot e^{-6t}\right)$$

$$= 2A_1 e^{-t} - \frac{1}{2} A_2 e^{-6t}$$

Saat $t = 0$:

$$\begin{array}{llll} A_1 \cdot & + A_2 \cdot & = 1 & \\ 2A_1 & - \frac{1}{2} A_2 \cdot & = \frac{5}{2} & \times 2 \end{array}$$

$$\begin{array}{lll} A_1 \cdot & + A_2 \cdot & = 1 \\ 4A_1 & - A_2 & = 5 \end{array} +$$

$$5A_1 = 6$$

$$A_1 = \frac{6}{5} = 1{,}2$$

$$A_2 = 1 - A_1 = -0{,}2$$

$$\therefore i = i_{L_2} = 1{,}2 e^{-t} - 0{,}2 e^{-6t}$$