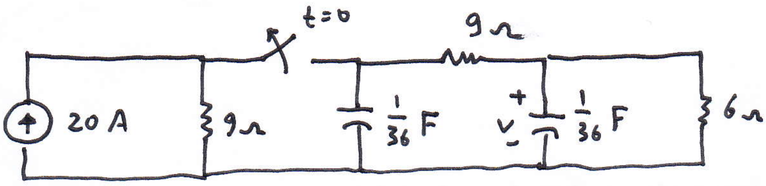
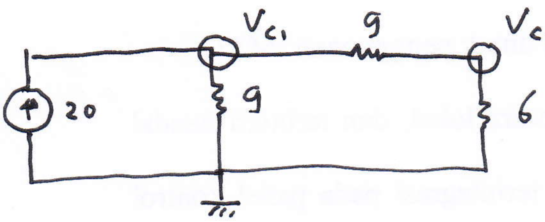
9.9 Find v for t > 0 if the circuit is in steady state at $t = 0^-$



Answer

Circuit at $t = 0^-$



Node V_{c_1}:

$$-20 + \frac{V_{c_1}}{9} + \frac{V_{c_1} - V_{c_2}}{9} = 0 \quad \times 9$$

$$2V_{c_1} - V_{c_2} = 180 \quad \ldots (1)$$

Node V_{c_2}:

$$\frac{V_{c_2} - V_{c_1}}{9} + \frac{V_{c_2}}{6} = 0 \quad \times 54$$

$$6V_{c_2} - 6V_{c_1} + 9V_{c_2} = 0$$

$$15V_{c_2} - 6V_{c_1} = 0 \quad \ldots (2)$$

(1) & (2):

$$\begin{array}{l|l} 2V_{c_1} - V_{c_2} = 180 & \times 3 \\ -6V_{c_1} + 15V_{c_2} = 0 & \times 1 \end{array}$$

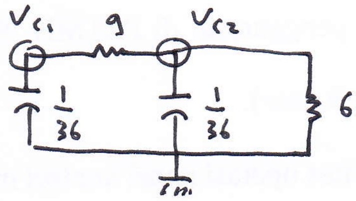
$$6V_{c_1} - 3V_{c_2} = 3 \cdot 180$$
$$-6V_{c_1} + 15V_{c_2} = 0 \quad +$$

$$12V_{c_2} = 3 \cdot 180$$

$$V_{c_2} = 45$$

$$V_{c_1} = \frac{180 + V_{c_2}}{2} = 112{,}5$$

Circuit at $t = 0^+$



Node V_{c_1}:

$$\frac{1}{36}\frac{dV_{c_1}}{dt} + \frac{V_{c_1} - V_{c_2}}{9} = 0 \quad \times 36$$

$$\frac{dV_{c_1}}{dt} + 4V_{c_1} - 4V_{c_2} = 0 \quad \ldots (3)$$

Node V_{c_2}:

$$\frac{V_{c_2} - V_{c_1}}{9} + \frac{1}{36}\frac{dV_{c_2}}{dt} + \frac{V_{c_2}}{6} = 0 \quad \times 36$$

$$4V_{c_2} - 4V_{c_1} + \frac{dV_{c_2}}{dt} + 6V_{c_2} = 0$$

$$4V_{c_1} = \frac{dV_{c_2}}{dt} + 10V_{c_2} \quad \ldots (4)$$

$$V_{c_1} = \frac{1}{4}\frac{dV_{c_2}}{dt} + \frac{10}{4}V_{c_2}$$

$$\frac{dV_{c_1}}{dt} = \frac{1}{4}\frac{d^2V_{c_2}}{dt^2} + \frac{10}{4}\frac{dV_{c_2}}{dt} \quad \ldots (5)$$

(3), (4), (5):

$$\left(\frac{1}{4}\frac{\partial^2 V_{c_2}}{dt^2} + \frac{10}{4}\frac{\partial V_{c_2}}{dt}\right) + \left(\frac{\partial V_{c_2}}{dt} + 10V_{c_2}\right) - 4V_{c_2} = 0$$

$$\frac{1}{4}\frac{\partial^2 V_{c_2}}{dt^2} + \frac{14}{4}\frac{\partial V_{c_2}}{dt} + 6V_{c_2} = 0 \quad \times 4$$

$$\frac{\partial^2 V_{c_2}}{dt^2} + 14\frac{\partial V_{c_2}}{dt} + 24 V_{c_2} = 0$$

transformasi:

$$s^2 + 14s + 24 = 0$$

$$(s+2)(s+12) = 0$$

$$s_1 = -2 \qquad s_2 = -12$$

$$V_{c_2} = A_1 e^{-2t} + A_2 e^{-12t} \quad \ldots (6)$$

$$V_{c_1} = \frac{1}{4}\frac{\partial V_{c_2}}{dt} + \frac{10}{4}V_{c_2}$$

$$= \frac{1}{4}\left(-2A_1 e^{-2t} - 12A_2 e^{-12t}\right) + \frac{10}{4}\left(A_1 e^{-2t} + A_2 e^{-12t}\right)$$

$$= 2A_1 e^{-2t} - \frac{1}{2}A_2 e^{-12t} \quad \ldots (7)$$

saat $t = 0$

$$A_1 + A_2 = 45 \quad | \times 1$$
$$2A_1 - \frac{1}{2}A_2 = 112{,}5 \quad | \times 2$$

$$A_1 + A_2 = 45$$
$$4A_1 - A_2 = 225 \quad +$$

$$A_1 = \frac{270}{5} = 54$$

$$A_2 = 45 - A_1 = -9$$

$$\therefore V = V_c = 54e^{-2t} - 9e^{-12t}$$