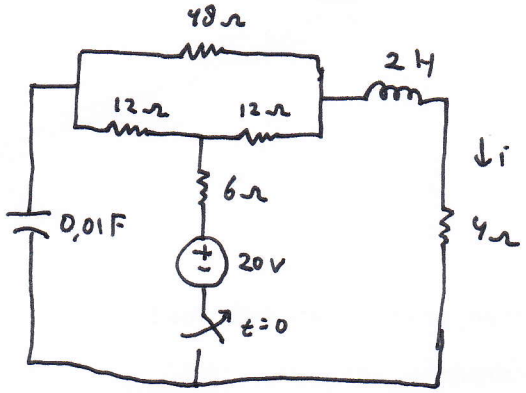
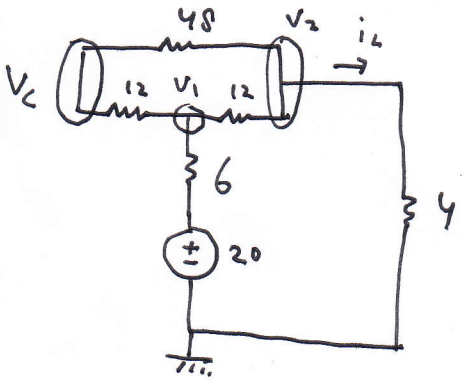
9.13 Find i for $t>0$ if the circuit is in steady state at $t=0^-$



Circuit at $t=0^-$



node V_c:

$$\frac{V_c-V_1}{12}+\frac{V_c-V_2}{48}=0 \quad \times 48$$

$$4V_c-4V_1+V_c-V_2=0$$

$$5V_c-4V_1-V_2=0 \quad \dots (1)$$

node V_1:

$$\frac{V_1-V_c}{12}+\frac{V_1-V_2}{12}+\frac{V_1-20}{6}=0 \quad \times 12$$

$$V_1-V_c+V_1-V_2+2V_1-40=0$$

$$4V_1-V_c-V_2=40 \quad \dots (2)$$

node V_2:

$$\frac{V_2-V_c}{48}+\frac{V_2-V_1}{12}+\frac{V_2}{4}=0 \quad \times 48$$

$$V_2-V_c+4V_2-4V_1+12V_2=0$$

$$17V_2-4V_1-V_c=0 \quad \dots (3)$$

(1) & (2):

$$-4V_1+5V_c-V_2=0$$
$$4V_1-V_c-V_2=40$$
$$+$$
$$4V_c-2V_2=40 \quad \dots (4)$$

(2) & (3)

$$4V_1-V_c-V_2=40$$
$$-4V_1-V_c+17V_2=0$$
$$+$$
$$-2V_c+16V_2=40 \quad \dots (5)$$

(4) & (5)

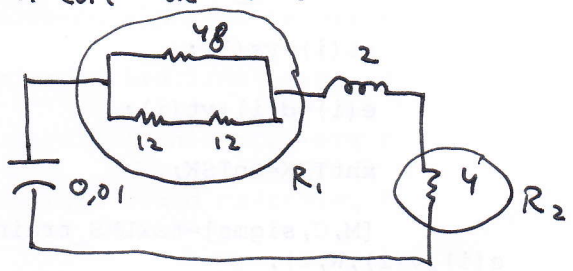
$$4V_c-2V_2=40 \quad | \times 8$$
$$-2V_c+16V_2=40 \quad | \times 1$$

$$32V_c-16V_2=320$$
$$-2V_c+16V_2=40$$
$$+$$
$$30V_c=360$$
$$V_c=12$$

$$V_2=\frac{4V_c-40}{2}=4$$

$$i_t=\frac{V_2}{4}=1\ A$$

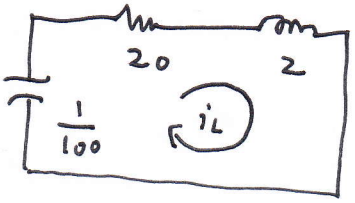
Circuit at $t=0^+$



$$R_1=\frac{(12+12)\times 48}{(12+12)+48}=\frac{24\times 48}{72_3}=16$$

$$R_1+R_2=20$$

Equivalent circuit:



loop i_L:

$$\frac{1}{0{,}01}\int i_L\,dt + K + 20 i_L + 2\frac{di_L}{dt} = 0$$

diff.

$$100\, i_L + 20\frac{di_L}{dt} + 2\frac{di_L}{dt} = 0 \quad \times \frac{1}{2}$$

$$\frac{d^2 i_L}{dt^2} + 10\frac{di_L}{dt} + 50 i_L = 0$$

transformasi:

$$s^2 + 10s + 50 = 0$$

$$s_{1,2} = \frac{-10 \pm \sqrt{100-200}}{2} = -5 \pm 5j$$

$$i_L = e^{-5t}(B_1 \sin(5t) + B_2 \cos(5t))$$

$$V_L = L\frac{di_L}{dt}$$

$$= 2\Big(-5e^{-5t}(B_1 \sin 5t + B_2 \cos 5t) + e^{-5t}(5B_1 \cos 5t - 5B_2 \sin 5t)\Big)$$

$$= 2\Big(-5B_1 e^{-5t}\sin 5t - 5B_2 e^{-5t}\cos 5t + -5B_2 e^{-5t}\sin 5t + 5B_1 e^{-5t}\cos 5t\Big)$$

$$= (-10B_1 - 10B_2)e^{-5t}\sin 5t + (-10B_2 + 10B_1)e^{-5t}\cos 5t$$

$$= -10(B_1 + B_2)e^{-5t}\sin 5t + 10(B_1 - B_2)e^{-5t}\cos 5t$$

$$V_C = -V_R - V_L$$

$$= -20(B_1 e^{-5t}\sin 5t + B_2 e^{-5t}\cos 5t) + 10(B_1 + B_2)e^{-5t}\sin 5t - 10(B_1 - B_2)e^{-5t}\cos 5t$$

at $t = 0$:

$$i_L = 1 = B_2$$

$$V_C = 12 = -20B_2 - 10(B_1 - B_2)$$

$$12 = -20B_2 + 10B_2 - 10B_1$$

$$12 = -10B_2 - 10B_1$$

$$B_1 = \frac{12 - 10}{10} = 0{,}2$$

$$\therefore i = e^{-5t}(0{,}2\sin 5t + \cos 5t)$$