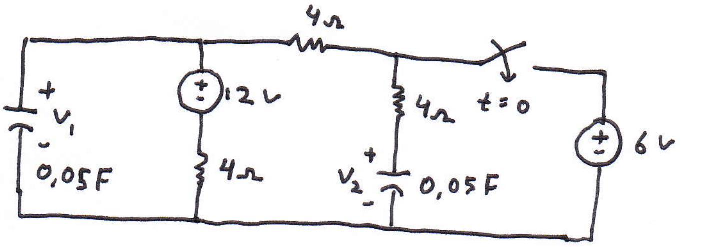
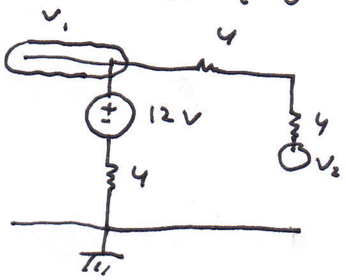
9.21 Find v_1 and v_2 for $t > 0$ if the circuit is in steady state at $t = 0^-$.



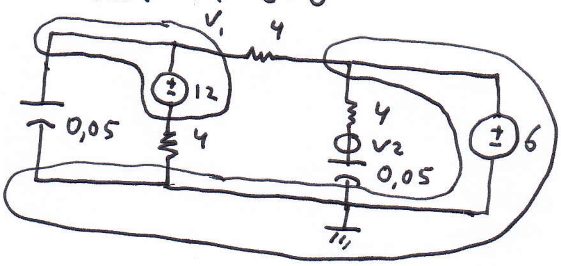
Circuit at $t = 0^-$



$v_1 = 12V$

$v_2 = 12V$

Circuit at $t = 0^+$



node V_1:

$$0{,}05\frac{dV_1}{dt} + \frac{V_1 - 12}{4} + \frac{V_1 - 6}{4} = 0 \qquad \times 4$$

$$0{,}2\frac{dV_1}{dt} + V_1 - 12 + V_1 - 6 = 0$$

$$0{,}2\frac{dV_1}{dt} + 2V_1 - 18 = 0 \qquad \times 5$$

$$\frac{dV_1}{dt} + 10V_1 - 90 = 0$$

$$\frac{dV_1}{dt} + 10V_1 = 90$$

General equation

$$\frac{dy}{dt} + Py = Q$$

$$y = e^{-Pt}\int Q\, e^{Pt}\,dt + Ae^{-Pt}$$

$$V_1 = e^{-10t}\int 90 \cdot e^{10t}\,dt + A\cdot e^{-10t}$$

$$V_1 = A\cdot e^{-10t} + \frac{90}{10}\cdot e^{-10t}\cdot e^{10t}$$

$$= A\cdot e^{-10t} + 9$$

at $t = 0$:

$$V_1 = 12 = A\cdot e^{-10t} + 9$$

$$A = 12 - 9 = 3$$

$$\therefore V_1 = 3e^{-10t} + 9$$

node V_2:

$$\frac{V_2 - 6}{4} + 0{,}05\frac{dV_2}{dt} = 0 \qquad \times 20$$

$$5V_2 - 30 + \frac{dV_2}{dt} = 0$$

$$\frac{dV_2}{dt} + 5V_2 = 30$$

$$V_2 = e^{-5t}\int 30\cdot e^{5t}\,dt + A\cdot e^{-5t}$$

$$= A\cdot e^{-5t} + 6$$

at $t = 0$

$$V_2 = 12 = A\cdot e^{-5t} + 6$$

$$A = 6$$

$$\therefore V_2 = 6\cdot e^{-5t} + 6$$