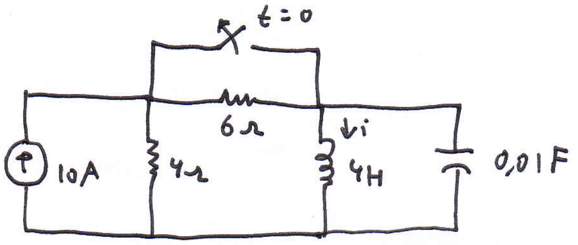
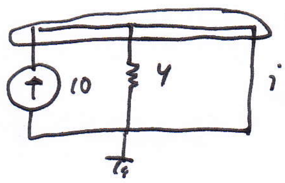
9.25 Find i for $t>0$ if the circuit is in steady state at $t=0^-$



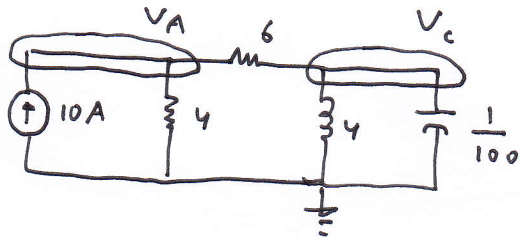
circuit at $t=0^-$



$i = 10A$

$V_c = 0$

Circuit at $t=0+$



Node V_A:

$$-10 + \frac{V_A}{4} + \frac{V_A - V_c}{6} = 0 \quad \times 12$$

$$3V_A + 2V_A - 2V_c = 120$$

$$V_A = \frac{120 + 2V_c}{5}$$

Node V_c:

$$\frac{V_c - V_A}{6} + \frac{1}{4}\int V_c\,dt + k + \frac{1}{100}\frac{dV_c}{dt} = 0 \quad \times 300$$

$$50V_c - 50V_A + 75\int V_c\,dt + k + 3\frac{dV_c}{dt} = 0$$

$$50V_c - 50\left(\frac{120+2V_c}{5}\right) + 75\int V_c\,dt + k + 3\frac{dV_c}{dt} = 0$$

$$50V_c - 1200 - 20V_c + 75\int V_c\,dt + k + 3\frac{dV_c}{dt} = 0$$

diff:

$$3\frac{\partial^2 V_c}{dt^2} + 30\frac{dV_c}{dt} + 75V_c = 0 \quad \times \frac{1}{3}$$

$$\frac{\partial^2 V_c}{dt^2} - 10\frac{dV_c}{dt} + 25V_c = 0$$

transf:

$$s^2 + 10s + 25 = 0$$

$$(s+5)(s+5) = 0$$

$$s_1 = s_2 = -5$$

$$V_{cN} = (A_1 + A_2 t)\cdot e^{-5t}$$

$$i_{LN} = \frac{1}{L}\int V_{cN}\,dt$$

$$\approx (C_1 + C_2 t)\cdot e^{-5t}$$

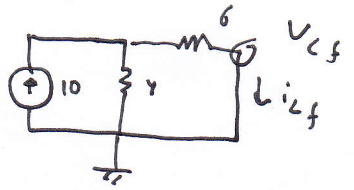
$$V_{LN} = L\frac{di_{LN}}{dt}$$

$$= 4\left(C_2\cdot e^{-5t} + -5(C_1 + C_2 t)e^{-5t}\right)$$

$$= 4C_2 e^{-5t} - 20C_1 e^{-5t} - 20C_2 t e^{-5t}$$

$$= (4C_2 - 20C_1 - 20C_2 t)e^{-5t}$$

forced response untuk constant forcing dapat diperoleh dari keadaan steady state :



$V_{cf} = 0$

$i_{Lf} = 10\cdot\frac{4}{4+6} = 4A$

complete response:

$$i_L = i_{LN} + i_{Lf}$$

$$= (C_1 + C_2 t)e^{-5t} + 4$$

$$V_L = V_{LN} + V_{Lf}$$

$$= (4C_2 - 20C_1 - 20C_2 t)e^{-5t}$$

at $t=0$:

$$i = 10 = C_1 + 4 \Leftrightarrow C_1 = 6$$

$$V_c = V_L = 0 = 4C_2 - 20\cdot 6$$

$$C_2 = \frac{\cancel{20}^{5}\cdot 6}{\cancel{4}_1} = 30$$

$$\therefore i = (6 + 30t)e^{-5t} + 4$$