As you can see, this is just the equation expressing the shifting property again.

Summary

In this chapter, we have introduced the z-transform of a sampled or discrete-time signal. The definition was given, and it was seen to be an algebraic equation. This definition was used to find the z-transform of an impulse signal and a sampled unit step signal. These two z-transforms will be used later to determine two digital filter approximations to analog filters. One other z-transform of a signal was determined, the z-transform of an exponential signal. This z-transform will be used later to determine the stability of digital filters.

We also determined one of the most important properties of z-transforms, the shifting property. This property will allow us to turn the difference equation of a digital filter into an algebraic equation and then determine the mathematical description of a digital filter, called its transfer function. Also, if we have determined the mathematical description, or transfer function, of a digital filter, we can use the shifting property to write the difference equation of the filter that is actually coded.

Table 4.1 gives the input signal into an ADC in column 1, the corresponding sampled signal in column 2, and the z-transform of that signal in column 3 for several signals using the definition of the z-transform. Table 4.1 can just as easily be used in the reverse direction to give the sampled time signal in column 2, given the z-transformed signal in column 3. This process will be needed in some of the following chapters.

Self-Test

1. Determine the z-transform of the following sampled signal.

$$x(n) = 5u(n)$$

2. Determine the z-transform of the following sampled signal.

$$y(n) = 3e^{-8n}u(n)$$

3. Determine the z-transform of the following analog signal after it goes through an ADC with $T = 0.01$ s.

$$x(t) = 10e^{-2t}u(t)$$

4. Determine the z-transform of the following sampled signal.

$$y(n) = 5(0.9)^n u(n)$$

5. Determine the z-transform of the following sampled signal.

$$x(n) = 1\delta(n-1) + 2\delta(n-2) + 3\delta(n-3)$$

6. Determine the z-transform of the following sampled signal.

$$x(n) = 5\cos(1.4n)u(n)$$

7. Determine the z-transform of the following analog signal after going through an ADC with $T = 0.01$ s.

$$x(t) = 21\sin(5t)u(t)$$

8. Determine the z-transform of the following sampled signal.

$$y(n) = 7(0.8)^n \cos(0.4n + 1.57)u(n)$$

9. Determine the z-transform of the following analog signal after going through an ADC with $T = 0.03$ s.

$$x(t) = 3tu(t)$$

10. Determine the z-transform of the following sampled signal.

$$x(n) = 0.34nu(n)$$

11. Determine the z-transform for the analog signal with the following Laplace transform after going through an ADC with the sampling period $T = 0.014$ s.

$$X(s) = \frac{2}{s}$$

12. Determine the z-transform for the analog signal with the following Laplace transform after going through an ADC with a sampling period $T = 0.002$ s.

$$X(s) = \frac{1.4}{s+5}$$

13. Determine the z-transform for the following sampled signal.

$$x(n) = 0.1u(n) - 0.1\delta(n) - 0.1\delta(n-1)$$

14. Determine the equation of the sampled pulse $x(n)$ described here, using the sampled unit step function.

$x(n)$ is 2 for $n = 0$ through and including $n = 5$
and zero for all other n's.

15. Determine the equation of the following sampled pulse signal.

$$x(n) = -3u(n) + 3u(n-7)$$

16. Given the only nonzero sample values of the following signal, determine the z-transform of the signal.

$$x(-1) = 2,\ x(0) = -1,\ x(2) = 1,\ x(3) = -4$$

17. Determine the z-transform of the following signal if it were sampled at $T = 0.05$ s.

$$x(t) = 3e^{-7t}\cos(25t)u(t)$$

Problems

1. Determine the z-transform of the following sampled signal.

$$x(n) = -6u(n)$$

2. Determine the z-transform of the following sampled signal.

$$y(n) = 5.7e^{-5n}u(n)$$

3. Determine the z-transform of the following analog signal after it goes through an ADC with $T = 0.02$ s.

$$x(n) = -7\sin(124t)u(t)$$

4. Determine the z-transform of the following sampled signal.

$$x(n) = 9(0.89)^n u(n)$$

5. Determine the z-transform of the following sampled signal.

$$y(n) = \delta(n) - 3\delta(n-1) + 2\delta(n-4)$$

6. Determine the z-transform of the following sampled signal.

$$x(n) = -4\sin(2.5n)u(n)$$

7. Determine the z-transform of the following analog signal after going through an ADC with $T = 0.07$ s.

$$x(n) = -5\cos(25t)u(t)$$

8. Determine the z-transform of the following sampled signal.

$$x(n) = 2(0.7)^n\cos(0.8n - 0.2)u(n)$$

9. Determine the z-transform of the following signal after going through an ADC with $T = 0.05$ s.

$$x(t) = 7tu(t)$$

10. Determine the z-transform of the following sampled signal.

$$y(n) = 1.37nu(n)$$

11. Determine the z-transform for the analog signal with the following Laplace transform after going through an ADC with the sampling period $T = 0.005$ s.

$$X(s) = \frac{-15}{s}$$

12. Determine the z-transform or the analog signal with the following Laplace transform after going through an ADC with a sampling period of T = 0.025 s.

$$X(x) = \frac{5}{s + 10}$$

13. Determine the z-transform for the following sampled signal.

$$y(n) = 2.4u(n - 1) + 4\delta(n)$$

14. Determine the equation of the following sampled pulse $x(n)$, using the sampled unit step function.

$x(n)$ is –3 for n = –1 through and including n = 4
and zero for all other n's.

15. Determine the equation of the following sampled pulse signal.

$$y(n) = 7u(n + 1) - 7u(n - 4)$$

16. Given the only nonzero values of the following signal, determine the z-transform of the signal.

$$x(-2) = -1,\ x(1) = 2,\ x(2) = -1$$

17. Determine the z-transform of the following signal if it were sampled at T - 0.1 s.

$$x(t) = -7e^{-3t}\cos(17t)u(t)$$

Answers to Self-Test

1. $\dfrac{5z}{z - 1}$

2. $\dfrac{3z}{z - e^{-8}}$

3. $\dfrac{10z}{z - e^{-0.02}}$

4. $\dfrac{5z}{z-0.9}$

5. $1z^{-1}+2z^{-2}+3z^{-3}$

6. $\dfrac{5z[z-\cos(1.4)]}{z^2-2z\cos(1.4)+1}$

7. $\dfrac{21z\sin(0.05)}{z^2-2\cos(0.05)+1}$

8. $\dfrac{7z[z\cos(1.57)-0.8\cos(1.57-0.4)]}{z^2-1.6z\cos(0.4)+0.64}$

9. $\dfrac{0.09z}{(z-1)^2}$

10. $\dfrac{0.34z}{(z-1)^2}$

11. $\dfrac{2z}{z-1}$

12. $\dfrac{1.4z}{z-\mathrm{e}^{0.01}}$

13. $\dfrac{0.1z^{-1}}{z-1}$

14. $x(n)=2u(n)-2u(n-6)$

15. $X(z)=\dfrac{-3z}{z-1}+\dfrac{3z^{-6}}{z-1}=\dfrac{3z(z^{-7}-1)}{z-1}$

16. $X(z)=2z-1+z^{-2}-4z^{-3}$

17. $X(z)=\dfrac{3z(1-\mathrm{e}^{-0.35}\cos(1.25))}{z^2-2\mathrm{e}^{-0.35}z\cos(1.25)+\mathrm{e}^{-0.7}}$