

**MATAKULIAH SISTEM DIGITAL**  
**PERTEMUAN III**  
**ALJABAR BOOLE &**  
**RANGKAIAN LOGIKA**

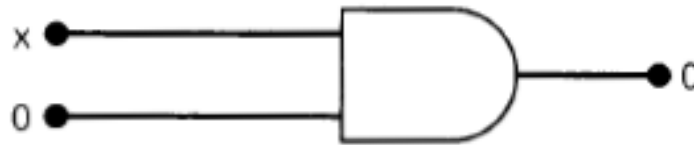
**OLEH :**  
**HIDAYAT**

**JURUSAN TEKNIK KOMPUTER**  
**UNIKOM**  
**2012**

# Teorema Boolean

variabel tunggal

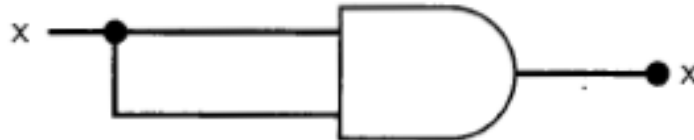
(1)  $x \cdot 0 = 0$



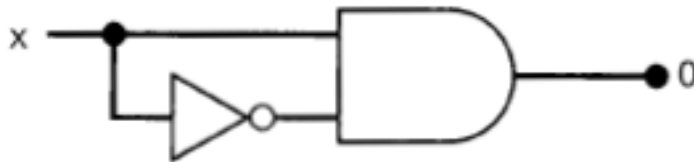
(2)  $x \cdot 1 = x$



(3)  $x \cdot x = x$



(4)  $x \cdot \bar{x} = 0$



# Teorema Boolean

variabel tunggal

(5)  $x + 0 = x$



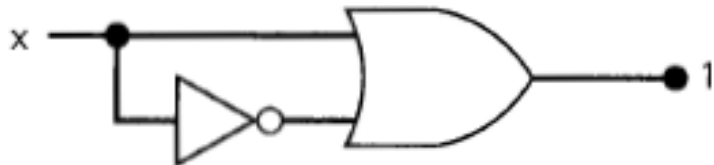
(6)  $x + 1 = 1$



(7)  $x + x = x$



(8)  $x + \bar{x} = 1$



# Teorema Boolean

variabel banyak (multivariabel)

$$(9) \quad x + y = y + x$$

$$(10) \quad x \cdot y = y \cdot x$$

$$(11) \quad x + (y + z) = (x + y) + z = x + y + z$$

$$(12) \quad x(yz) = (xy)z = xyz$$

$$(13a) \quad x(y + z) = xy + xz$$

$$(13b) \quad (w + x)(y + z) = wy + xy + wz + xz$$

$$(14) \quad x + xy = x$$

$$(15a) \quad x + \bar{x}y = x + y$$

$$(15b) \quad \bar{x} + xy = \bar{x} + y$$

# Teorema Boolean

variabel banyak (multivariabel) – pembuktian teorema 14

$$\begin{aligned}x + xy &= x \\0 + 0 \cdot 0 &= 0 \\0 &= 0\end{aligned}$$

$$\begin{aligned}x + xy &= x \\0 + 0 \cdot 1 &= 0 \\0 + 0 &= 0 \\0 &= 0\end{aligned}$$

$$\begin{aligned}x + xy &= x(1 + y) \\&= x \cdot 1 \\&= x\end{aligned}$$

$$\begin{aligned}x + xy &= x \\1 + 1 \cdot 0 &= 1 \\1 + 0 &= 1 \\1 &= 1\end{aligned}$$

$$\begin{aligned}x + xy &= x \\1 + 1 \cdot 1 &= 1 \\1 + 1 &= 1 \\1 &= 1\end{aligned}$$

# Teorema Boolean

variabel banyak (multivariabel) – Contoh:

Simplify the expression  $y = A\bar{B}D + A\bar{B}\bar{D}$ .

$$y = A\bar{B}(D + \bar{D})$$

$$\begin{aligned} y &= A\bar{B} \cdot 1 \\ &= A\bar{B} \end{aligned}$$

Simplify  $z = (\bar{A} + B)(A + B)$ .

$$z = \bar{A} \cdot A + \bar{A} \cdot B + B \cdot A + B \cdot B$$

$$z = 0 + \bar{A} \cdot B + B \cdot A + B = \bar{A}B + AB + B$$

$$z = B(\bar{A} + A + 1)$$

$$z = B$$

# Teorema Boolean

## DeMorgan

$$(16) \quad \overline{(x + y)} = \bar{x} \cdot \bar{y}$$

$$(17) \quad \overline{(x \cdot y)} = \bar{x} + \bar{y}$$

$$\overline{x + y + z} = \bar{x} \cdot \bar{y} \cdot \bar{z}$$

$$\overline{x \cdot y \cdot z} = \bar{x} + \bar{y} + \bar{z}$$

*Example 1*

$$\begin{aligned} z &= \overline{A + \overline{B \cdot C}} \\ &= \bar{A} \cdot \overline{(\bar{B} \cdot \bar{C})} \\ &= \bar{A} \cdot (\bar{\bar{B}} + \bar{\bar{C}}) \\ &= \bar{A} \cdot (B + \bar{C}) \end{aligned}$$

*Example 2*

$$\begin{aligned} \omega &= \overline{(A + BC) \cdot (D + EF)} \\ &= \overline{(A + BC)} + \overline{(D + EF)} \\ &= (\bar{A} \cdot \overline{BC}) + (\bar{D} \cdot \overline{EF}) \\ &= [\bar{A} \cdot (\bar{B} + \bar{C})] + [\bar{D} \cdot (\bar{E} + \bar{F})] \\ &= \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{D}\bar{E} + \bar{D}\bar{F} \end{aligned}$$

# Bentuk Ekspresi Boolean



# SOP dan POS

- Ada dua bentuk persamaan/ekspresi Boolean:
  - Sum of Product (SOP) dan
  - Product of Sum (POS)
- Ekspresi SOP: terdiri dari dua atau lebih AND term (product) yang di**OR**kan. Contoh:

$$1. ABC + \bar{A}B\bar{C}$$

$$2. AB + \bar{A}B\bar{C} + \bar{C}\bar{D} + D$$

- Ekspresi POS: terdiri dari dua atau lebih OR term (sum) yang di**AND**kan. Contoh:

$$1. (A + \bar{B} + C)(A + C)$$

$$2. (A + \bar{B})(\bar{C} + D)F$$

$$3. (A + C)(B + \bar{D})(\bar{B} + C)(A + \bar{D} + \bar{E})$$

# SOP

- *Sum of Product* (SOP): terdiri dari dua atau lebih AND term (product) yang di**OR**kan.
- Setiap AND term/perkalian variabelnya disebut '**minterm**'
- Setiap *minterm* diperoleh dari fungsi output yang berlogika '1'.
- cara menuliskannya: sesuai variabelnya langsung jika variabel tersebut berlogika '1' dan menuliskan komplementnya jika berlogika '0'.

# POS

- *Product of Sum* (POS). Setiap penjumlahan variabelnya disebut '**maksterm**'.
- ekspresi boolean *maxterm* : terdiri dari dua atau lebih OR term (sum) yang di**AND**kan.
- Setiap *maxterm* diperoleh dari fungsi output yang berlogika 'o'
- Cara menuliskannya: mengkomplemenkan variabelnya jika variabel tersebut berlogika '1' dan menuliskannya sesuai variabelnya jika berlogika 'o'.

# SOP & POS 3 Variabel

A	B	C	minterm	maxterm
0	0	0	$m_0 = \overline{A}\overline{B}\overline{C}$	$M_0 = A + B + C$
0	0	1	$m_1 = \overline{A}\overline{B}C$	$M_1 = A + B + \overline{C}$
0	1	0	$m_2 = \overline{A}B\overline{C}$	$M_2 = A + \overline{B} + C$
0	1	1	$m_3 = \overline{A}BC$	$M_3 = A + \overline{B} + \overline{C}$
1	0	0	$m_4 = A\overline{B}\overline{C}$	$M_4 = \overline{A} + B + C$
1	0	1	$m_5 = A\overline{B}C$	$M_5 = \overline{A} + B + \overline{C}$
1	1	0	$m_6 = ABC\overline{C}$	$M_6 = \overline{A} + \overline{B} + C$
1	1	1	$m_7 = ABC$	$M_7 = \overline{A} + \overline{B} + \overline{C}$

# Contoh SOP:

A	B	C	Y	Perkalian Dasar
0	0	0	0	
0	0	1	1	$m_1 = \overline{A}\overline{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$m_4 = A\overline{B}\overline{C}$
1	0	1	1	$m_5 = A\overline{B}C$
1	1	0	1	$m_6 = AB\overline{C}$
1	1	1	0	

$$Y = m_1 + m_4 + m_5 + m_6$$

$$Y = \sum m(1, 4, 5, 6)$$

$$Y = \overline{A}\overline{B}C + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C}$$



Sederhanakan persamaannya dan gambarkan rangkaian logikanya !

# Contoh POS :

A	B	C	Y	Maxterm
0	0	0	0	$M_0 = A + B + C$
0	0	1	1	
0	1	0	0	$M_2 = A + \bar{B} + C$
0	1	1	0	$M_3 = A + \bar{B} + \bar{C}$
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	0	$M_7 = \bar{A} + \bar{B} + \bar{C}$

$$Y = M_0 + M_2 + M_3 + M_7$$

$$Y = \prod M(0, 2, 3, 7)$$

$$Y = (A + B + C) \cdot (A + \bar{B} + C) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B} + \bar{C})$$



Sederhanakan persamaannya dan gambarkan rangkaian logikanya !



# Penyederhanaan Rangkaian Logika

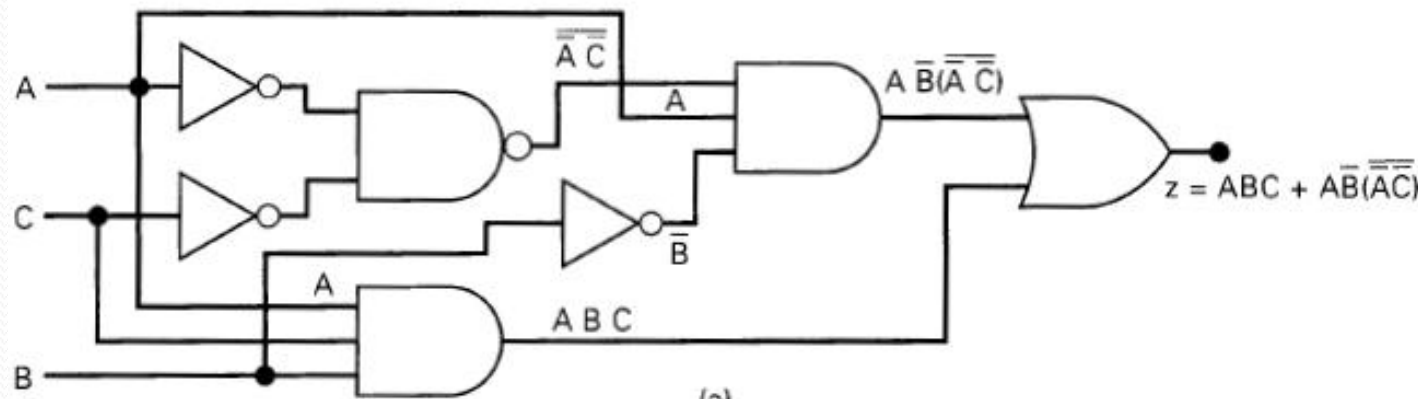
# Metoda Penyederhanaan

## Menggunakan teorema Aljabar Boolean

- ekspresi awal diambil dalam bentuk SOP dengan menggunakan teorema DeMorgan dan perkalian term untuk menghasilkannya.
- setelah itu, lakukan pemaktoran (jika memungkinkan) untuk mengeliminasi term yang mungkin dieliminasi.

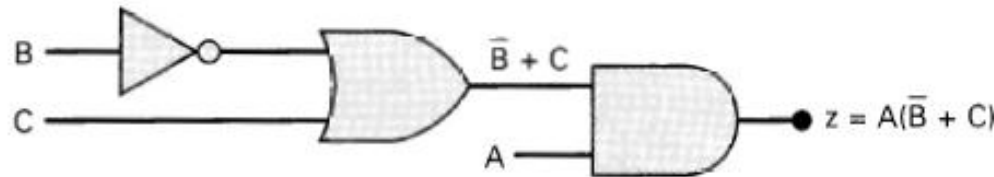
# Metoda Penyederhanaan

Menggunakan teorema Aljabar Boolean - **Contoh**



(a)

=



(b)

Caranya?

# Metoda Penyederhanaan

## Menggunakan teorema Aljabar Boolean - Contoh

$$z = ABC + A\bar{B} \cdot (\overline{\bar{A}\bar{C}})$$

$$z = ABC + A\bar{B}(\bar{A} + \bar{C})$$

[theorem (17)]

$$= ABC + A\bar{B}(A + C)$$

[cancel double inversions]

$$= ABC + A\bar{B}A + A\bar{B}C$$

[multiply out]

$$= ABC + A\bar{B} + A\bar{B}C$$

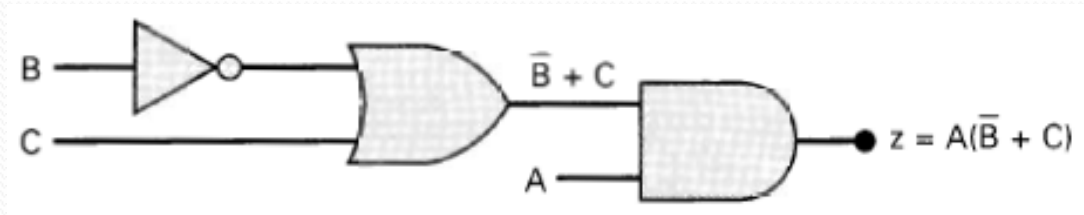
[ $A \cdot A = A$ ]

$$z = AC(B + \bar{B}) + A\bar{B}$$

$$z = AC(1) + A\bar{B}$$

$$= AC + A\bar{B}$$

$$z = A(C + \bar{B})$$



# Metoda Penyederhanaan

## Menggunakan teorema Aljabar Boolean – Contoh 2

$$z = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC$$

$$\begin{aligned} z &= \overline{A}\overline{B}(C + \overline{C}) + \overline{A}C(\overline{B} + B) \\ &= \overline{A}\overline{B} \cdot 1 + \overline{A}C \cdot 1 \\ &= \overline{A}\overline{B} + \overline{A}C = \overline{A}(\overline{B} + C) \end{aligned}$$

Simplify  $z = \overline{A}C(\overline{A}\overline{B}\overline{D}) + \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C$ .

$$z = \overline{A}C(A + \overline{B} + \overline{D}) + \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C$$

$$z = \overline{A}CA + \overline{A}C\overline{B} + \overline{A}C\overline{D} + \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C$$

$$z = \overline{A}\overline{B}C + \overline{A}C\overline{D} + \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C$$

$$z = \overline{B}C + \overline{A}\overline{D}(B + C)$$

Cara lain:

$$z = C(\overline{A}\overline{B} + \overline{A}\overline{D} + \overline{A}\overline{B}) + \overline{A}\overline{B}\overline{C}\overline{D}$$

$$z = C(\overline{B}[\overline{A} + A] + \overline{A}\overline{D}) + \overline{A}\overline{B}\overline{C}\overline{D}$$

$$z = C(\overline{B} + \overline{A}\overline{D}) + \overline{A}\overline{B}\overline{C}\overline{D}$$

$$z = \overline{B}C + \overline{A}C\overline{D} + \overline{A}\overline{B}\overline{C}\overline{D}$$

$$z = \overline{B}C + \overline{A}\overline{D}(C + \overline{B}\overline{C})$$

$$z = \overline{B}C + \overline{A}\overline{D}(B + C)$$

# Contoh Disain

Design a logic circuit that has three inputs,  $A$ ,  $B$ , and  $C$ , and whose output will be HIGH only when a majority of the inputs are HIGH.

Langkah 1: Susun tabel kebenaran

$A$	$B$	$C$	$x$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1 $\rightarrow \bar{A}BC$
1	0	0	0
1	0	1	1 $\rightarrow A\bar{B}C$
1	1	0	1 $\rightarrow AB\bar{C}$
1	1	1	1 $\rightarrow ABC$

Langkah 2: Tuliskan AND term untuk  $x = '1'$ .

Langkah 3: Tuliskan Ekspresi SOP untuk  $x$ .

$$x = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

Langkah 4: Sederhanakan ekspresinya.

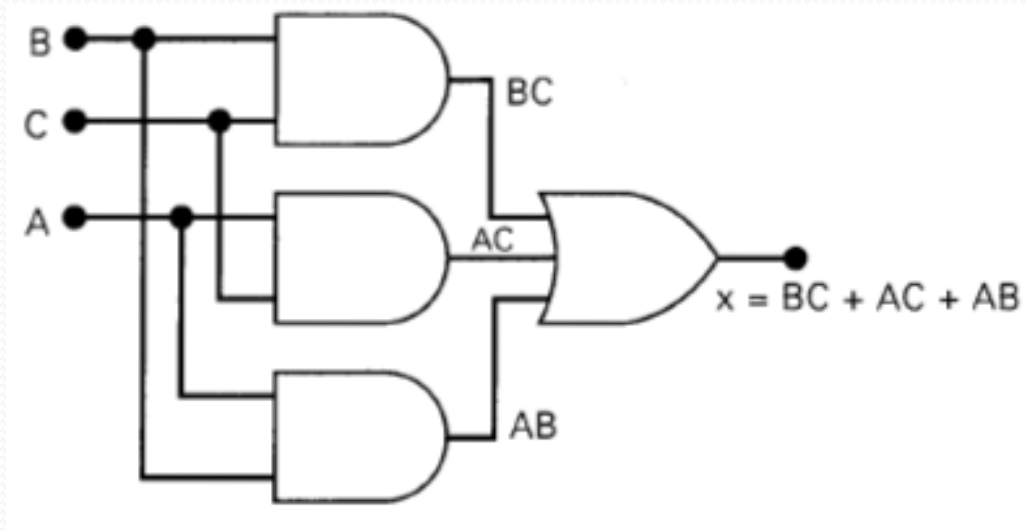
$$\begin{aligned} x &= \bar{A}BC + ABC + A\bar{B}C + ABC + AB\bar{C} + ABC \\ x &= BC(\bar{A} + A) + AC(\bar{B} + B) + AB(\bar{C} + C) \end{aligned}$$

$$x = BC + AC + AB$$

# Contoh Disain Lanj.

Langkah 5: Gambarkan rangkaian logikanya untuk ekspresi Boolean terakhir.

$$x = BC + AC + AB$$



# Metoda Penyederhanaan

## Menggunakan Peta Karnaugh

Banyaknya kotak pada peta karnaugh sesuai dengan banyaknya kemungkinan dalam tabel kebenaran, yaitu ' $2^n$ ' dengan  $n$  adalah banyaknya variabel

- Dua variabel  $\rightarrow$  4 kotak
- Tiga variabel  $\rightarrow$  8 kotak
- Empat Variabel  $\rightarrow$  16 kotak



# Dua variabel

Tabel Kebenaran 2 variabel

A	B	F
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

K-Map 2 variabel

		B	$\bar{B}$	
		0	1	
A	0	$m_0$	$m_1$	$\bar{A}$
	1	$m_2$	$m_3$	A

# Dua variabel

Tabel Kebenaran 2 variabel

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

K-Map 2 variabel

		B		
		0	1	
A	0	1	0	$\bar{A}$
	1	1	0	A

Persamaan sederhana yang didapat ?

# Tiga variabel

Tabel Kebenaran 3 variabel

A	B	C	F
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

K-Map 3 variabel

		C		
		$\bar{C}$ 0	C 1	
B	$\bar{B}$ 00	$m_0$	$m_1$	$\bar{A}$
	01	$m_2$	$m_3$	
	11	$m_6$	$m_7$	A
	$\bar{B}$ 10	$m_4$	$m_5$	

# Tiga variabel

Tabel Kebenaran 3 variabel

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

K-Map 3 variabel

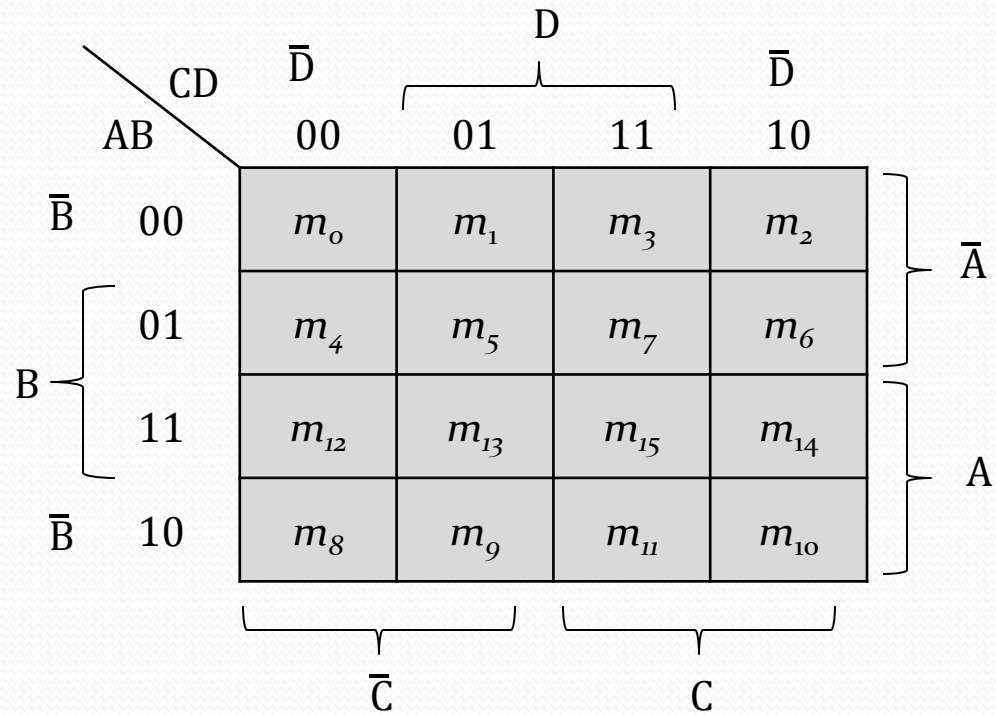
		C	
		$\bar{C}$	C
B	AB	0	1
	$\bar{B}$ 00	1	1
	01	0	0
	11	0	0
$\bar{B}$	10	1	1

Diagram illustrating the K-Map 3 variabel. The map is a 4x2 grid of cells. The columns are labeled  $\bar{C}$  and C. The rows are labeled AB (00, 01, 11, 10). The cells contain values: (00,  $\bar{C}$ )=1, (00, C)=1, (01,  $\bar{C}$ )=0, (01, C)=0, (11,  $\bar{C}$ )=0, (11, C)=0, (10,  $\bar{C}$ )=1, (10, C)=1. Brackets on the right indicate that the first two rows (00 and 01) are grouped under  $\bar{A}$ , and the last two rows (11 and 10) are grouped under A.

Persamaan sederhana yang didapat ?

# Empat variabel

A	B	C	D	F
0	0	0	0	$m_0$
0	0	0	1	$m_1$
0	0	1	0	$m_2$
0	0	1	1	$m_3$
0	1	0	0	$m_4$
0	1	0	1	$m_5$
0	1	1	0	$m_6$
0	1	1	1	$m_7$
1	0	0	0	$m_8$
1	0	0	1	$m_9$
1	0	1	0	$m_{10}$
1	0	1	1	$m_{11}$
1	1	0	0	$m_{12}$
1	1	0	1	$m_{13}$
1	1	1	0	$m_{14}$
1	1	1	1	$m_{15}$



# Empat variabel

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

		D					
		$\bar{D}$	$D$		$\bar{D}$		
CD	AB	00	01	11	10		
	$\bar{B}$	00	01	11	10		
B	$\bar{B}$	00	1	1	0	0	$\bar{A}$
	$\bar{B}$	01	1	1	1	1	
	$\bar{B}$	11	0	0	1	1	A
	$\bar{B}$	10	1	0	0	0	
		$\bar{C}$		C			

Persamaan sederhana yang didapat ?

# Penyederhanaan pada K-MAP

- Pengelompokkan nilai '1' yang saling berdekatan.
  - *pairs* (pasangan),
  - *quads* dan
  - *octet*.

# Pairs

- Akan menghilangkan sebuah variabel

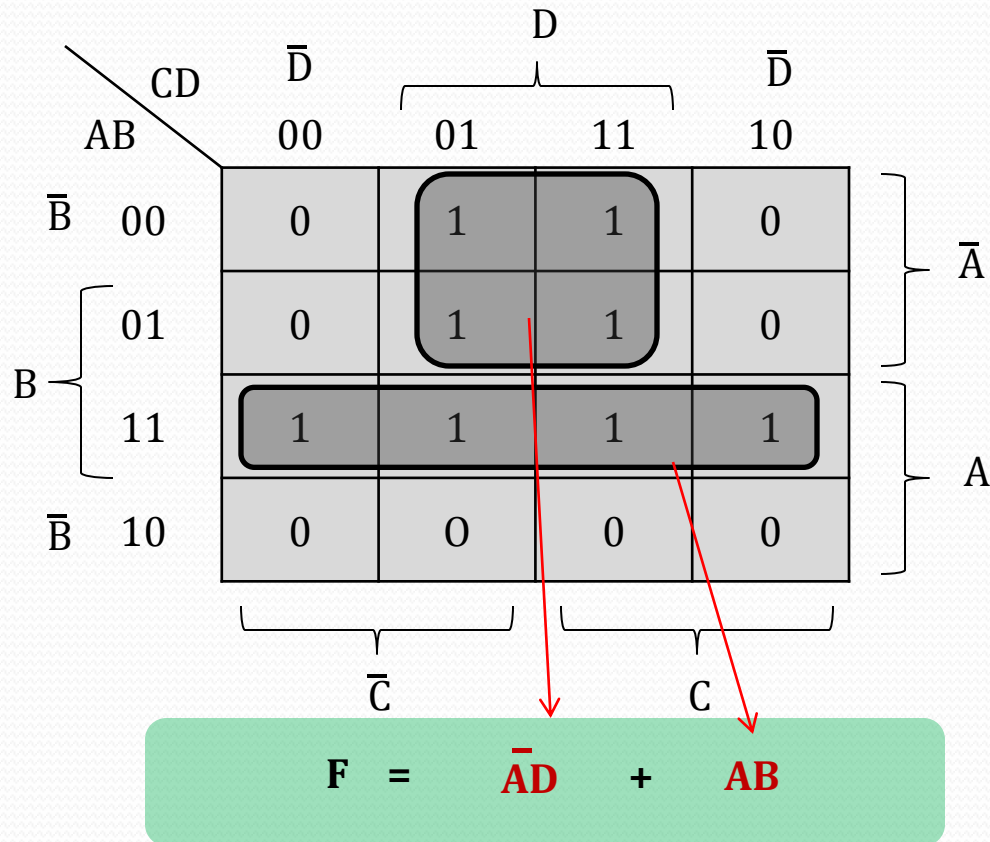
		D				
		$\bar{D}$			$\bar{D}$	
		00	01	11	10	
AB	$\bar{B}$ 00	0	1	1	0	$\bar{A}$
	01	1	0	0	0	
	11	1	0	1	1	A
	$\bar{B}$ 10	0	0	0	0	

$$F = \bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{D} + ABC$$



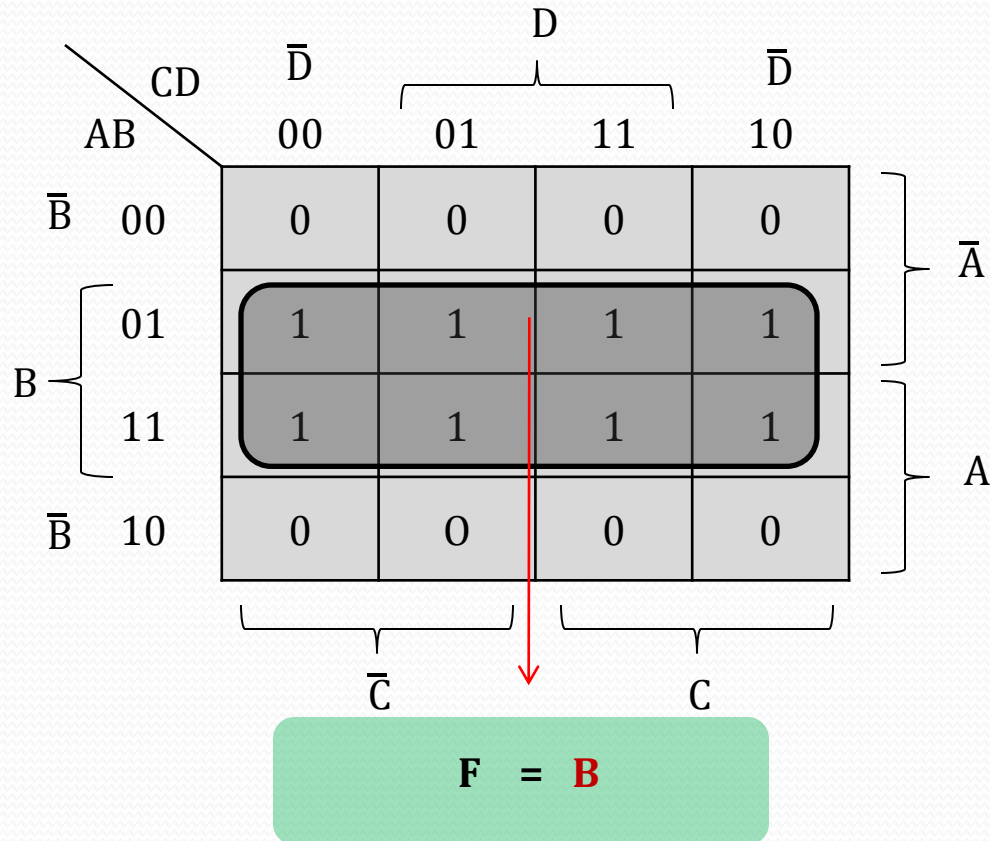
# Quads

- Akan menghilangkan 2 buah variabel



# Octet

- Akan menghilangkan 3 buah variabel



# Overlapping

- Mengelompokkan logika '1' yg sama lebih dari 1 klpmk

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	1	0
10	0	0	0	0

$$F = ABC\bar{C} + BD$$

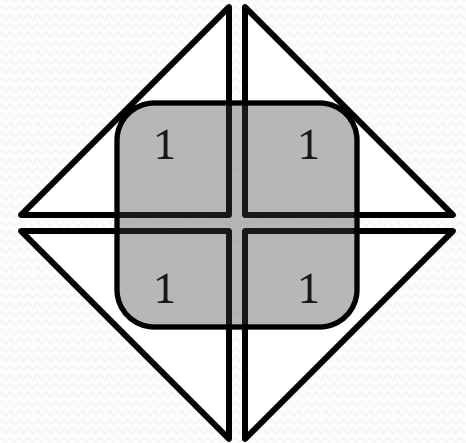
# Rolling

- Mengelompokkan logika '1' dg cara penggulungan.

CD		00	01	11	10
AB	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

-

$$F = \bar{B}\bar{D}$$



# Rolling

- Mengelompokkan logika '1' dg cara penggulungan.

CD		00	01	11	10
AB	00	1	1	1	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	1	1	1

1	1	1	1
1	1	1	1

$$F = \bar{B}$$

# Redundant

- Kelompok berlebih

CD		00	01	11	10
AB	00	0	0	0	0
01	0	0	1	1	0
11	1	1	0	0	0
10	0	0	0	0	0

$$F = \textcolor{red}{AB\bar{C}} + \textcolor{red}{B\bar{C}D} + \textcolor{red}{\bar{A}BD}$$

# Don't care

CD \ AB		CD			
		00	01	11	10
00	0	0	0	x	x
01	0	0	0	1	x
11	0	x		1	1
10	0	0	0	x	x


$$F = C$$

# Konklusi

- isikan nilai '1' pada peta Karnaugh minterm dg nilai '1' pada tabel kebenaran. Selanjutnya yang bernilai '0'.
- lingkari oktet, quad dan pairs. Ingat roll dan overlap untuk memperluas pengelompokan
- jika ada sisa bernilai '1' lingkari
- hilangkan kelompok yang berlebihan
- tuliskan persamaan Boolean dengan meng OR kan perkalian dari kelompok lingkaran



*“Man lam yarkabi al ahwaala lam yanaali al amaala”*

Siapa yang tidak mengalami kesukaran, ia tidak akan mencapai apa yang dicita-citakannya.